HALF RANGE SERIES:

COSINE SERIES:

The half range cosine series in the interval $(0,l)$ is given by $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{l}x$.

Where $a_0 = \frac{2}{l}\int_0^l f(x)\,dx$; $a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi}{l}x\,dx$.

SINE SERIES:

The half range sine series in the interval $(0,l)$ is given by $f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi}{l}x$.

where $b_n = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi}{l}x\,dx$.

1) Find the Fourier sine series expansion for $f(x)=x$ in $(0,l)$

Soln: Let $f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi}{l}x$.

Now $b_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi}{l}\right)x\,dx$

$$= \frac{2}{l}\int_0^l x\sin\left(\frac{n\pi}{l}\right)x\,dx.$$

$$= \frac{2}{l}\left[x\left(\frac{-\cos(n\pi/l)x}{(n\pi/l)}\right) - (1)\left(\frac{-\sin(n\pi/l)x}{(n\pi/l)^2}\right)\right]_0^l$$

$$= \frac{2}{l}\left[-l\cdot\frac{\cos n\pi}{n\pi/l}\right]$$

$$= -\frac{2l}{n\pi}(-1)^n = \frac{2l}{n\pi}(-1)^{n+1}$$

$$\therefore f(x) = \sum_{n=1}^{\infty}\frac{2l}{n\pi}(-1)^{n+1}\sin\left(\frac{n\pi}{l}\right)x.$$

3) Obtain the Fourier cosine series for $f(x) = \begin{cases} kx, & 0 \le x \le l/2 \\ k(l-x), & l/2 \le x \le l. \end{cases}$

Soln:

Let $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{l}x$.

Now $a_0 = \frac{2}{l}\int_0^l f(x)\,dx$.

$$= \frac{2}{l}\left\{\int_0^{l/2} kx\,dx + \int_{l/2}^{l} k(l-x)\,dx\right\}$$

$$= \frac{2k}{l}\left\{\left[\frac{x^2}{2}\right]_0^{l/2} + \left[lx - \frac{x^2}{2}\right]_{l/2}^{l}\right\}$$

$$= \frac{2k}{l}\left\{\frac{l^2}{8} + \left[\frac{l^2}{2} - \left(\frac{l^2}{2} - \frac{l^2}{8}\right)\right]\right\}$$

$$= \frac{2k}{l}\cdot\frac{2l^2}{8}$$

$$= \frac{kl}{2}$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi}{l}x\,dx$$

$$= \frac{2}{l}\left\{\int_0^{l/2} kx\cos\frac{n\pi}{l}x\,dx + \int_{l/2}^{l} k(l-x)\cos\frac{n\pi}{l}x\,dx\right\}$$

$$= \frac{2k}{l}\left\{\left[x\left(\frac{\sin\left(\frac{n\pi}{l}\right)x}{\left(\frac{n\pi}{l}\right)}\right) - 1\left(\frac{-\cos\left(\frac{n\pi}{l}\right)x}{\left(\frac{n\pi}{l}\right)^2}\right)\right]_0^{l/2} + \left[(l-x)\left(\frac{\sin\left(\frac{n\pi}{l}\right)x}{\left(\frac{n\pi}{l}\right)}\right) - (-1)\left(-\frac{\cos\left(\frac{n\pi}{l}\right)x}{\left(\frac{n\pi}{l}\right)^2}\right)\right]_{l/2}^{l}\right\}$$

$$= \frac{2k}{l}\left\{\left[\frac{l}{2}\frac{\sin n\pi/2}{\left(\frac{n\pi}{l}\right)} + \frac{\cos n\pi/2}{(n\pi/l)^2} - \frac{\cos 0}{(n\pi/l)^2}\right] + \left[0 - \frac{\cos n\pi}{\left(\frac{n\pi}{l}\right)^2} - \left(\frac{l}{2}\frac{\cancel{\sin n\pi/2}}{\left(\frac{n\pi}{l}\right)} - \frac{\cos n\pi/2}{(n\pi/l)^2}\right)\right]\right\}$$

$$= \frac{2k}{l}\left\{\frac{\cos n\pi/2}{(n\pi/l)^2} - \frac{1}{(n\pi/l)^2} - \frac{(-1)^n}{(n\pi/l)^2} + \frac{\cos n\pi/2}{(n\pi/l)^2}\right\}$$

$$= \frac{2k}{l}\left[\frac{2\cos n\pi/2}{(n\pi/l)^2} - \frac{[1+(-1)^n]}{(n\pi/l)^2}\right]$$

$$= \frac{2kl}{n^2\pi^2}\left[2\cos n\pi/2 - [1+(-1)^n]\right]$$

$$\therefore f(x) = \frac{kl}{4} + \sum_{n=1}^{\infty}\frac{2kl}{n^2\pi^2}\left[2\cos n\pi/2 - (1+(-1)^n)\right]\cos\left(\frac{n\pi}{l}\right)x$$

3) Find the Fourier series for cosine for $f(x) = x$ in $(0, l)$