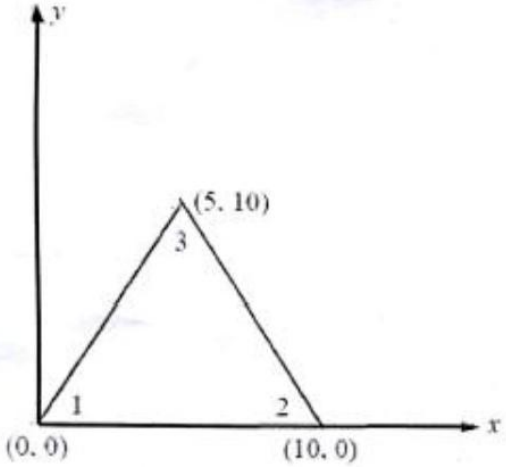
Evaluate the stiffness matrix for the elements shown in Figure 1. The coordinates are given in units of millimeters. Assume plane stress conditions. Let E =210GPa, $\mu = 0.25$,and t = 10 mm.



$(x_1, y_1) = (0,0)$

$(x_2, y_2) = (10,0)$

$(x_3, y_3) = (5,10)$

To Find:-

$$[k] = [B]^T [D][B] \cdot A \cdot t$$

Step i) Area of triangular Element.

$$A = \frac{1}{2}\begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} \Rightarrow A = \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 10 & 0 \\ 1 & 5 & 10 \end{bmatrix}$$

$$= \frac{1}{2}\left[1(10\times 10 - 0) - 0 + 0\right]$$

$$A = \frac{100}{2} = 50 mm^2$$

Step ii) Strain Displacement matrix

$$B = \frac{1}{2A}\begin{bmatrix} q_1 & 0 & q_2 & 0 & q_3 & 0 \\ 0 & r_1 & 0 & r_2 & 0 & r_3 \\ r_1 & q_1 & r_2 & q_2 & r_3 & q_3 \end{bmatrix}$$

$\therefore q_1 = y_2 - y_3 = 0 - 10 = -10$

$q_2 = y_3 - y_1 = 10 - 0 = 10$

$q_3 = y_1 - y_2 = 0 - 0 = 0$

$r_1 = x_3 - x_2 = 5 - 10 = -5$

$r_2 = x_1 - x_3 = 0 - 5 = -5$

$r_3 = x_2 - x_1 = 10 - 0 = 10$

$$[B] = \frac{1}{2 \times 50} \times 5 \begin{bmatrix} 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -2 & -1 & 2 & 2 & 0 \end{bmatrix}$$

$$[B] = \frac{1}{20} \begin{bmatrix} -2 & 0 & 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -2 & -1 & 2 & 2 & 0 \end{bmatrix}$$

$$[B]^T = \frac{1}{20} \begin{bmatrix} -2 & 0 & -1 \\ 0 & -1 & -2 \\ 2 & 0 & -1 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

Stress strain matrix [D] for plane stress condition:

$$[D] = \frac{E}{1-\mu^2} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix}$$

$$[D] = \frac{2.1 \times 10^5}{1 - 0.25^2} \begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & \frac{1-0.25}{2} \end{bmatrix}$$

$$[D] = \frac{2.1 \times 10^5}{1 - 0.25^2} [0.25] \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

$$[D] = 56000 \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

$$[k] = [B]^T [D] [B] \cdot A \cdot t$$

$$= [D] [B]$$

$$= 56 \times 10^3 \times \frac{1}{20} \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix} \times$$

$$\begin{bmatrix} -2 & 0 & 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -2 & -1 & 2 & 2 & 0 \end{bmatrix}$$

$$= 2800 \begin{bmatrix} -8 & -1 & 8 & -1 & 0 & 2 \\ -2 & -4 & 2 & -4 & 0 & 8 \\ -1.5 & -3 & -1.5 & 3 & 3 & 0 \end{bmatrix}$$

To find:

$$[k] = [B]^T [D] [B] \cdot A \cdot t$$

$$= [B]^T [D] [B]$$

$$= \frac{2800}{2} \begin{bmatrix} -2 & 0 & -1 \\ 0 & -1 & -2 \\ 2 & 0 & -1 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} -8 & -1 & 8 & -1 & 0 & 2 \\ -2 & -4 & 2 & -4 & 0 & 8 \\ -1.5 & -3 & -1.5 & 3 & 3 & 0 \end{bmatrix}$$

$$= 70000 \begin{bmatrix} 17.5 & 5 & -14.5 & -1 & -3 & -1 \\ 5 & 10 & 1 & -2 & -6 & -8 \\ -14.5 & 1 & 17.5 & -5 & -3 & 1 \\ -1 & -2 & -5 & 10 & 6 & -7 \\ -3 & -6 & -3 & 6 & 6 & 0 \\ -1 & -8 & 1 & -8 & 0 & 16 \end{bmatrix}$$