3. ***Successfully constrained motion.*** When the motion between the elements, forming a pair, is such that the constrained motion is not completed by itself, but by some other means, then the motion is said to be successfully constrained motion. Consider a shaft in a foot-step bearing as shown in Figure 5. The shaft may rotate in a bearing or it may move upwards. This is a case of incompletely constrained motion. But if the load is placed on the shaft to prevent axial upward movement of the shaft, then the motion of the pair is said to be successfully constrained motion. The motion of an I.C. engine valve (these are kept on their seat by a spring) and the piston reciprocating inside an engine cylinder are also the examples of successfully constrained motion.

Classification of Kinematic Pairs

The kinematic pairs may be classified according to the following considerations:

1. ***According to the type of relative motion between the elements.*** The kinematic pairs according to type of relative motion between the elements may be classified as discussed below:

(*a*) ***Sliding pair.*** When the two elements of a pair are connected in such a way that one can only slide relative to the other, the pair is known as a sliding pair. The piston and cylinder, cross-head and guides of a reciprocating steam engine, ram and its guides in shaper, tail stock on the lathe bed etc. are the examples of a sliding pair. A little consideration will show, that a sliding pair has a completely constrained motion.

(*b*) ***Turning pair***. When the two elements of a pair are connected in such a way that one can only turn or revolve about a fixed axis of another link, the pair is known as turning pair. A shaft with collars at both ends fitted into a circular hole, the crankshaft in a journal bearing in an engine, lathe spindle supported in head stock, cycle wheels turning over their axles etc. are the examples of turning pair. A turning pair also has a completely constrained motion.

(*c*) ***Rolling pair***. When the two elements of a pair are connected in such a way that one rolls over another fixed link, the pair is known as rolling pair. Ball and roller bearings are examples of rolling pair.

(*d*) ***Screw pair.*** When the two elements of a pair are connected in such a way that one element can turn about the other by screw threads, the pair is known as screw pair. The lead screw of a lathe with nut, and bolt with a nut are examples of a screw pair.

(*e*) ***Spherical pair***. When the two elements of a pair are connected in such a way that one element (with spherical shape) turns or swivels about the other fixed element, the pair formed is called a spherical pair. The ball and socket joint, attachment of a car mirror, pen stand etc., are the examples of a spherical pair.

2. ***According to the type of contact between the elements***. The kinematic pairs according to the type of contact between the elements may be classified as discussed below :

(*a*) ***Lower pair***. When the two elements of a pair have a surface contact when relative motion takes place and the surface of one element slides over the surface of the other, the pair formed is known as lower pair. It will be seen that sliding pairs, turning pairs and screw pairs form lower pairs.

(*b*) ***Higher pair***. When the two elements of a pair have a line or point contact when relative motion takes place and the motion between the two elements is partly turning and partly sliding, then the pair is known as higher pair. A pair of friction discs, toothed gearing, belt and rope drives, ball and roller bearings and cam and follower are the examples of higher pairs.

3. ***According to the type of closure***. The kinematic pairs according to the type of closure

between the elements may be classified as discussed below:

(*a*) ***Self closed pair***. When the two elements of a pair are connected together mechanically in such a way that only required kind of relative motion occurs, it is then known as self-closed pair. The lower pairs are self-closed pair.

(*b*) ***Force - closed pair***. When the two elements of a pair are not connected mechanically but are kept in contact by the action of external forces, the pair is said to be a force-closed pair. The cam and follower is an example of force closed pair, as it is kept in contact by the forces exerted by spring and gravity.

Kinematic Chain

When the kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion (*i.e.* completely or successfully constrained motion), it is called a ***kinematic chain.*** In other words, a kinematic chain may be defined as a combination of kinematic pairs, joined in such a way that each link forms a part of two pairs and the relative motion between the links or elements is completely or successfully constrained. For example, the crankshaft of an engine forms a kinematic pair with the bearings which are fixed in a pair, the connecting rod with the crank forms a second kinematic pair, the piston with the connecting rod forms a third pair and the piston with the cylinder forms a fourth pair. The total combination of these links is a kinematic chain.

If each link is assumed to form two pairs with two adjacent links, then the relation between the number of pairs (p) forming a kinematic chain and the number of links (l) may be expressed in the form of an equation:

$$l = 2p - 4 \quad \ldots (i)$$

Since in a kinematic chain each link forms a part of two pairs, therefore there will be as many links as the number of pairs. Another relation between the number of links (l) and the number of joints (j) which constitute a kinematic chain is given by the expression:

$$J = 3/2\, l - 2 \quad \ldots (ii)$$

The equations ***(i)*** and ***(ii)*** are applicable only to kinematic chains, in which lower pairs are used. These equations may also be applied to kinematic chains, in which higher pairs are used. In that case each higher pair may be taken as equivalent to two lower pairs with an additional element or link.

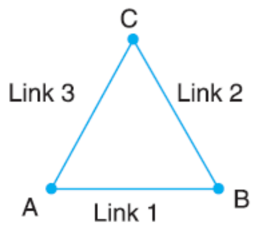
Let us apply the above equations to the following cases to determine whether each of them is a kinematic chain or not.

Consider the arrangement of three links *AB*, *BC* and *CA* with pin joints at *A*, *B* and *C* as shown in Figure. In this case,

Number of links, $l = 3$

Number of pairs, $p = 3$

and number of joints, $j = 3$



From equation **(*i*)**, $l = 2p - 4$

or $3 = 2 \times 3 - 4 = 2$

L.H.S. > R.H.S.

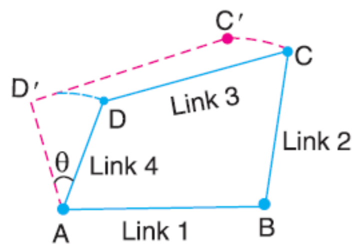
Now from equation **(*ii*),**

$$j = \frac{3}{2}\, l - 2 \quad \text{or} \quad 3 = \frac{3}{2} \times 3 - 2 = 2.5$$

L.H.S. > R.H.S.

Since the arrangement of three links, as shown in Figure, does not satisfy the equations **(*i*)** and **(*ii*)** and the left-hand side is greater than the right-hand side, therefore it is not a kinematic chain and hence no relative motion is possible. Such type of chain is called ***locked chain*** and forms a rigid frame or structure which is used in bridges and trusses.

2. Consider the arrangement of four links *AB*, *BC*, *CD* and *DA* as shown in Figure. In this case



$l = 4$, $p = 4$, and $j = 4$

From equation **(*i*),** $l = 2\,p - 4$

$4 = 2 \times 4 - 4 = 4$

i.e. L.H.S. = R.H.S.

From equation **(*ii*),**