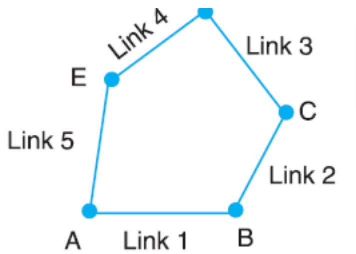
$$j = \frac{3}{2}l - 2$$

$$4 = \frac{3}{2} \times 4 - 2 = 4$$

Since the arrangement of four links, as shown in Figure, satisfy the equations **(*i*)** and **(*ii*),**

therefore, it is a ***kinematic chain of one degree of freedom***. A chain in which a single link such as *AD* in Figure is sufficient to define the position of all other links, it is then called a kinematic chain of one degree of freedom. A little consideration will show that in Figure, if a definite displacement (say θ) is given to the link *AD*, keeping the link *AB* fixed, then the resulting displacements of the remaining two links *BC* and *CD* are also perfectly definite. Thus, we see that in a four-bar chain, the relative motion is completely constrained. Hence it may be called as a ***constrained kinematic chain***, and it is the basis of all machines.

3. Consider an arrangement of five links, as shown in Figure. In this case,



$l = 5$, $p = 5$, and $j = 5$

From equation **(*i*),**

$l = 2p - 4$ or $5 = 2 \times 5 - 4 = 6$

i.e. L.H.S. < R.H.S.

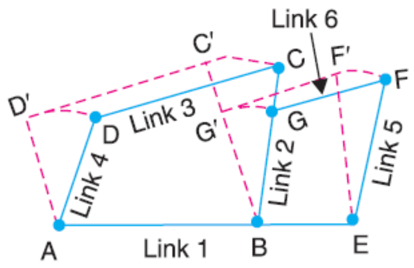
From equation **(*ii*),**

$$j = \frac{3}{2} l - 2 \quad \text{or} \quad 5 = \frac{3}{2} \times 5 - 2 = 5.5$$

Since the arrangement of five links, as shown in Figure does not satisfy the equations and left-hand side is less than right hand side, therefore it is not a kinematic chain. Such a type of chain is called ***unconstrained chain*** *i.e*. the relative motion is not completely constrained. This type of chain is of little practical importance.

4. Consider an arrangement of six links, as shown in Figure. This chain is formed by adding two more links in such a way that these two links form a pair with the existing links as well as form themselves a pair. In this case

l = 6, p = 5, and j = 7



From equation ***(i)*,**

$l = 2\,p - 4$ or $6 = 2 \times 5 - 4 = 6$

i.e. L.H.S. = R.H.S.

From equation ***(ii)*,**

$$j = \frac{3}{2}\,l - 2 \quad \text{or} \quad 7 = \frac{3}{2} \times 6 - 2 = 7$$

i.e. L.H.S. = R.H.S.

Since the arrangement of six links, as shown in Figure, satisfies the equations (*i.e.* left-hand side is equal to right hand side), therefore it is a kinematic chain.

Note: A chain having more than four links is known as **compound kinematic chain.**

Types of Joints in a Chain

The following types of joints are usually found in a chain:

1. *Binary joint*. When two links are joined at the same connection, the joint is known as binary joint. For example, a chain as shown in Figure, has four links and four binary joins at *A*, *B*, *C* and *D*.

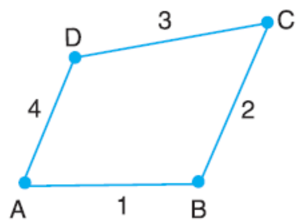


Figure: Kinematic chain with all binary joints.

In order to determine the nature of chain, *i.e*. whether the chain is a locked chain (or structure) or kinematic chain or unconstrained chain, the following relation between the number of links and the number of binary joints, as given by A.W. Klein, may be used :

$$j + \frac{h}{2} = \frac{3}{2}l - 2$$ ... ***(i)***

where j = Number of binary joints,

h = Number of higher pairs, and

l = Number of links.

When h = 0, the equation ***(i)***, may be written as

$$j = \frac{3}{2}l - 2$$ . . . ***(ii)***

Applying this equation to a chain, as shown in Figure, where l = 4 and j = 4, we have

$$4 = \frac{3}{2} \times 4 - 2 = 4$$

Since the left-hand side is equal to the right-hand side, therefore the chain is a kinematic chain or constrained chain.

2. ***Ternary joint***. When three links are joined at the same connection, the joint is known as ternary joint. It is equivalent to two binary joints as one of the three links joined carry the pin for the other two links. For example, a chain, as shown in Figure, has six links. It has three binary joints at *A*, *B* and *D* and two ternary joints at *C* and *E*. Since one ternary joint is equivalent to two binary joints, therefore equivalent binary joints in a chain, as shown in Figure, are 3 + 2 × 2 = 7

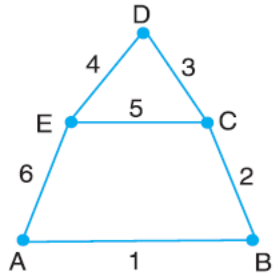


Figure. Kinematic chain having binary and ternary joints.

Let us now determine whether this chain is a kinematic chain or not. We know that $l = 6$ and $j = 7$, therefore from equation ***(ii)*,**

$$j = \frac{3}{2} l - 2$$

$$7 = \frac{3}{2} \times 6 - 2 = 7$$

Since left hand side is equal to right hand side, therefore the chain, as shown in Figure, is a kinematic chain or constrained chain.

3. *Quaternary joint.* When four links are joined at the same connection, the joint is called a quaternary joint. It is equivalent to three binary joints. In general, when l number of links are joined at the same connection, the joint is equivalent to $(l - 1)$ binary joints.

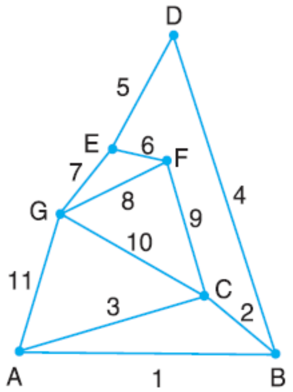


Figure (a) Looked chain having binary, ternary and quaternary joints.

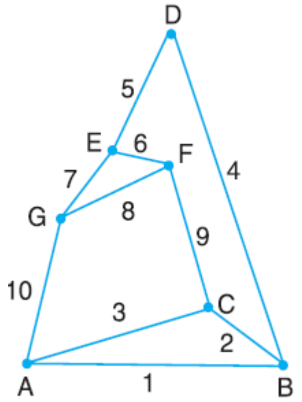


Figure (b) Kinematic chain having binary and ternary joints.

For example, consider a chain having eleven links, as shown in Figure (a). It has one binary joint at D, four ternary joints at A, B, E and F, and two quaternary joints at C and G. Since one quaternary joint is equivalent to three binary joints and one ternary joint is equal to two binary joints, therefore total number of binary joints in a chain, as shown in Figure (a), are

$$1 + 4 \times 2 + 2 \times 3 = 15$$

Let us now determine whether the chain, as shown in Figure (*a*), is a kinematic chain or not. We know that $l = 11$ and $j = 15$. We know that,

$$j = \frac{3}{2} l - 2, \quad \text{or} \quad 15 = \frac{3}{2} \times 11 - 2 = 14.5, \ i.e., \text{ L.H.S.} > \text{R.H.S.}$$

Since the left hand side is greater than right hand side, therefore the chain, as shown in Figure (*a*) , is not a kinematic chain. We have discussed in Art 5.9, that such a type of chain is called locked chain and forms a rigid frame or structure. If the link *CG* is removed, as shown in Figure (*b*), it has ten links and has one binary joint at *D* and six ternary joints at *A*, *B*, *C*, *E*, *F* and *G*. Therefore, total number of binary joints are 1 + 2 × 6 = 13. We know that

$$j = \frac{3}{2} l - 2, \quad \text{or} \quad 13 = \frac{3}{2} \times 10 - 2 = 13, \ i.e. \text{ L.H.S.} = \text{R.H.S.}$$

Since left hand side is equal to right hand side, therefore the chain, as shown in Figure (*b*), is a kinematic chain or constrained chain.