Mechanism

When one of the links of a kinematic chain is fixed, the chain is known as ***mechanism***. It may be used for transmitting or transforming motion *e.g.* engine indicators, typewriter etc.

A mechanism with four links is known as ***simple mechanism***, and the mechanism with more than four links is known as ***compound mechanism***. When a mechanism is required to transmit power or to do some particular type of work, it then becomes a ***machine***. In such cases, the various links or elements have to be designed to withstand the forces (both static and kinetic) safely.

A little consideration will show that a mechanism may be regarded as a machine in which each part is reduced to the simplest form to transmit the required motion.

Number of Degrees of Freedom for Plane Mechanisms

In the design or analysis of a mechanism, one of the most important concerns is the number of degrees of freedom (also called movability) of the mechanism. It is defined as the number of input parameters (usually pair variables) which must be independently controlled in order to bring the mechanism into a useful engineering purpose. It is possible to determine the number of degrees of freedom of a mechanism directly from the number of links and the number and types of joints which it includes. Consider a four-bar chain, as shown in Figure (*a*). A little consideration will show that only one variable such as θ is needed to define the relative positions of all the links. In other words, we say that the number of degrees of freedom of a four-bar chain is one. Now, let us consider a five-bar chain, as shown in Figure (*b*). In this case two variables such as θ_1 and θ_2 are needed to define completely the relative positions of all the links. Thus, we say that the number of degrees of freedom is * two.

In order to develop the relationship in general, consider two links *AB* and *CD* in a plane motion as shown in Figure (*a*).

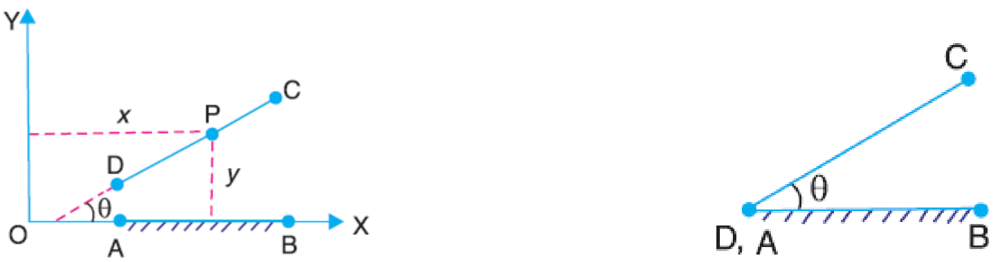

Figure (a) and (b). Links in a plane motion.

The link *AB* with co-ordinate system *OXY* is taken as the reference link (or fixed link). The position of point *P* on the moving link *CD* can be completely specified by the three variables, *i.e*. the co-ordinates of the point *P* denoted by *x* and *y* and the inclination θ of the link *CD* with X-axis or link *AB*. In other words, we can say that each link of a mechanism has three degrees of freedom before it is connected to any other link. But when the link *CD* is connected to the link *AB* by a turning pair at *A*, as shown in Figure (*b*), the position of link *CD* is now determined by a single variable θ and thus has one degree of freedom. From above, we see that when a link is connected to a fixed link by a turning pair (*i.e*. lower pair), two degrees of freedom are destroyed. This may be clearly understood from Figure, in which the resulting four bar mechanism has one degree of freedom (*i.e*. $n = 1$).

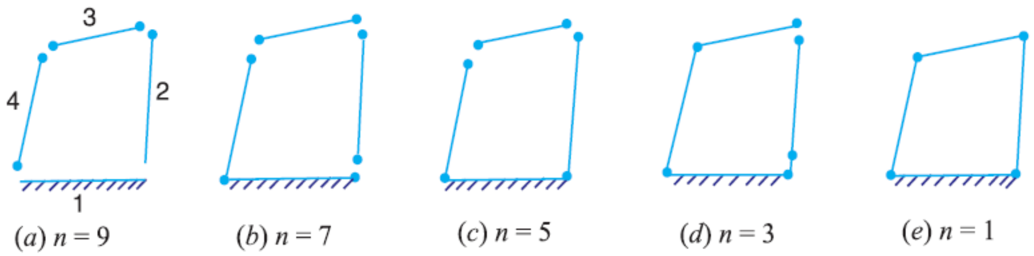

Figure. Four bar mechanism.

Now let us consider a plane mechanism with l number of links. Since in a mechanism, one of the links is to be fixed, therefore the number of movable links will be (l – 1) and thus the total number of degrees of freedom will be 3 (l – 1) before they are connected to any other link. In general, a mechanism with l number of links connected by j number of binary joints or lower pairs (i.e. single degree of freedom pairs) and h number of higher pairs (i.e. two degree of freedom pairs), then the number of degrees of freedom of a mechanism is given by

$$n = 3\,(l - 1) - 2\,j - h \quad \text{... }(i)$$

This equation is called Kutzbach criterion for the movability of a mechanism having plane motion. If there are no two degree of freedom pairs (*i.e*. higher pairs), then $h = 0$. Substituting $h = 0$ in equation (*i*), we have

$$n = 3\,(l - 1) - 2\,j \quad \text{... }(ii)$$

Application of Kutzbach Criterion to Plane Mechanisms

We have discussed in the previous article that Kutzbach criterion for determining the number of degrees of freedom or movability (n) of a plane mechanism is

$$n = 3 (l - 1) - 2j - h$$

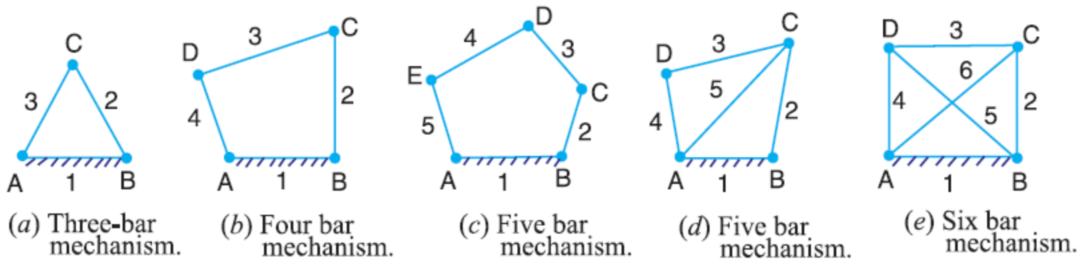

Figure. Plane mechanisms.

The number of degrees of freedom or movability (n) for some simple mechanisms having no higher pair (*i.e.* $h = 0$), as shown in Figure, are determined as follows:

1. The mechanism, as shown in Figure (a), has three links and three binary joints, *i.e.* $l = 3$ and $j = 3$.

   $n = 3 (3 - 1) - 2 \times 3 = 0$

2. The mechanism, as shown in Fig (b), has four links and four binary joints, *i.e.* $l = 4$ and $j = 4$.

   $n = 3 (4 - 1) - 2 \times 4 = 1$

3. The mechanism, as shown in Fig. 5.16 (c), has five links and five binary joints, *i.e.* $l = 5$, and $j = 5$.

   $n = 3 (5 - 1) - 2 \times 5 = 2$

4. The mechanism, as shown in Figure (d), has five links and six equivalent binary joints (because there are two binary joints at B and D, and two ternary joints at A and C), *i.e.* $l = 5$ and $j = 6$.

   $n = 3 (5 - 1) - 2 \times 6 = 0$

5. The mechanism, as shown in Fig. 5.16 (e), has six links and eight equivalent binary joints (because there are four ternary joints at A, B, C and D), *i.e.* $l = 6$ and $j = 8$.

   $n = 3 (6 - 1) - 2 \times 8 = -1$

It may be noted that

(*a*) When $n = 0$, then the mechanism forms a structure and no relative motion between the links is possible, as shown in Figure (*a*) and (*d*).

(*b*) When $n = 1$, then the mechanism can be driven by a single input motion, as shown in Figure (*b*).

(*c*) When $n = 2$, then two separate input motions are necessary to produce constrained motion for the mechanism, as shown in Figure (*c*).

(*d*) When $n = -1$ or less, then there are redundant constraints in the chain and it forms a statically indeterminate structure, as shown in Figure (*e*).

The application of Kutzbach's criterion applied to mechanisms with a higher pair or two degree of freedom joints is shown in Figure.

Figure. Mechanism with a higher pair.

In Figure (*a*), there are three links, two binary joints and one higher pair, *i.e.* $l = 3$, $j = 2$ and $h = 1$.

$$\therefore n = 3(3-1) - 2 \times 2 - 1 = 1$$

In Figure (*b*), there are four links, three binary joints and one higher pair, *i.e.* $l = 4$, $j = 3$ and $h = 1$

$$\therefore n = 3(4-1) - 2 \times 3 - 1 = 2$$

Here it has been assumed that the slipping is possible between the links (*i.e.* between the wheel and the fixed link). However, if the friction at the contact is high enough to prevent slipping, the joint will be counted as one degree of freedom pair, because only one relative motion will be possible between the links.