Solution. Given: CD = 50 mm; CA = 75 mm; PA = 150 mm; PR = 135 mm;

The extreme positions of the driving crank are shown in Figure. From the geometry of the figure,

$$\cos\beta/2 = \frac{CD}{CA_2} = \frac{50}{75} = 0.667 \qquad \ldots (\because CA_2 = CA)$$

$$\beta/2 = 48.2° \quad \text{or} \quad \beta = 96.4°$$

Ratio of the time of cutting stroke to the time of return stroke

We know that

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{360-\beta}{\beta} = \frac{360-96.4}{96.4} = 2.735 \quad \textbf{Ans.}$$

Length of effective stroke

In order to find the length of effective stroke (*i.e.* R_1R_2), draw the space diagram of the mechanism to some suitable scale, as shown in Figure. Mark $P_1R_2 = P_2R_2 = PR$. Therefore, by measurement we find that,

Length of effective stroke = R_1R_2 = 87.5 mm **Answer.**

Double Slider Crank Chain

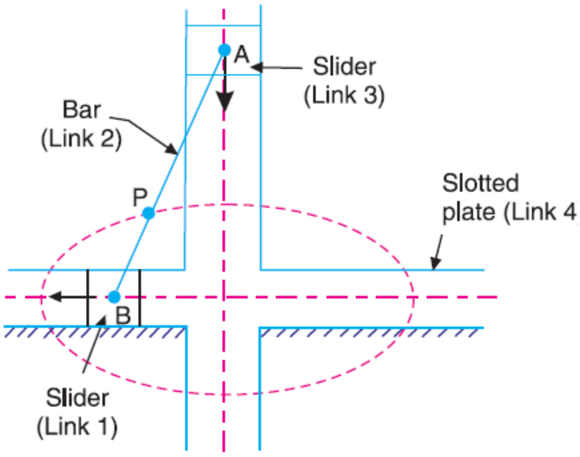
A kinematic chain which consists of two turning pairs and two sliding pairs is known as ***double slider crank chain***, as shown in Figure. We see that the link 2 and link 1 form one turning pair and link 2 and link 3 form the second turning pair. The link 3 and link 4 form one sliding pair and link 1 and link 4 form the second sliding pair.

Inversions of Double Slider Crank Chain

The following three inversions of a double slider crank chain are important from the subject point of view:

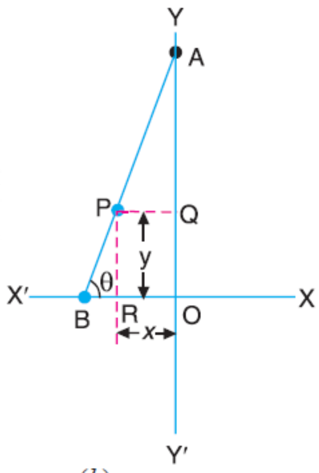
1. ***Elliptical trammels***. It is an instrument used for drawing ellipses. This inversion is obtained by fixing the slotted plate (link 4), as shown in Figure. The fixed plate or link 4 has two straights' grooves cut in it, at right angles to each other. The link 1 and link 3, are known as sliders and form sliding pairs with link 4. The link AB (link 2) is a bar which forms turning pair with links 1 and 3. When the links 1 and 3 slide along their respective grooves, any point on the link 2 such as P traces out an ellipse on the surface

of link 4, as shown in Figure (*a*). A little consideration will show that *AP* and *BP* are the semi-major axis and semi-minor axis of the ellipse respectively. This can be proved as follows:



(*a*)

Figure. Elliptical trammels.



(*b*)

Figure. Elliptical trammels

Let us take OX and OY as horizontal and vertical axes and let the link BA is inclined at an angle θ with the horizontal, as shown in Figure (b). Now the co-ordinates of the point P on the link BA will be

$$x = PQ = AP\cos\theta;\ \text{and}\ y = PR = BP\sin\theta$$

$$\frac{x}{AP} = \cos\theta;\ \text{and}\ \frac{y}{BP} = \sin\theta$$

Squaring and adding,

$$\frac{x^2}{(AP)^2} + \frac{y^2}{(BP)^2} = \cos^2\theta + \sin^2\theta = 1$$

This is the equation of an ellipse. Hence the path traced by point *P* is an ellipse whose semimajor axis is *AP* and semi-minor axis is *BP*.

2. ***Scotch yoke mechanism***. This mechanism is used for converting rotary motion into a reciprocating motion. The inversion is obtained by fixing either the link 1 or link 3. In Figure, link1 is fixed. In this mechanism, when the link 2 (which corresponds to crank) rotates about *B* as centre, the link 4 (which corresponds to a frame) reciprocates. The fixed link 1 guides the frame.

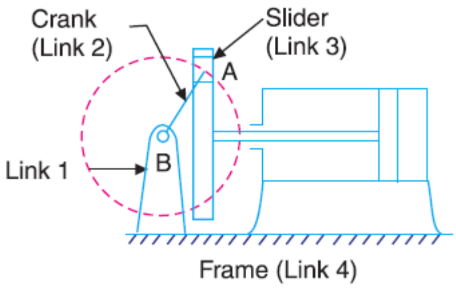


Figure. Scotch yoke mechanism.

3. ***Oldham's coupling.*** An oldham's coupling is used for connecting two parallel shafts whose axes are at a small distance apart. The shafts are coupled in such a way that if one shaft rotates, the other shaft also rotates at the same speed. This inversion is obtained by fixing the link 2, as shown in Figure (a). The shafts to be connected have two flanges (link 1 and link 3) rigidly fastened at their ends by forging. The link 1 and link 3 form turning pairs with link 2. These flanges have diametrical slots cut in their inner faces, as shown in Figure (b). The intermediate piece (link 4) which is a circular disc, have two tongues (i.e. diametrical projections) T1 and T2 on each face at right angles to each other, as shown in Figure (c). The tongues on the link 4 closely fit into the slots in the two flanges (link 1 and link 3). The link 4 can slide or reciprocate in the slots in the flanges.

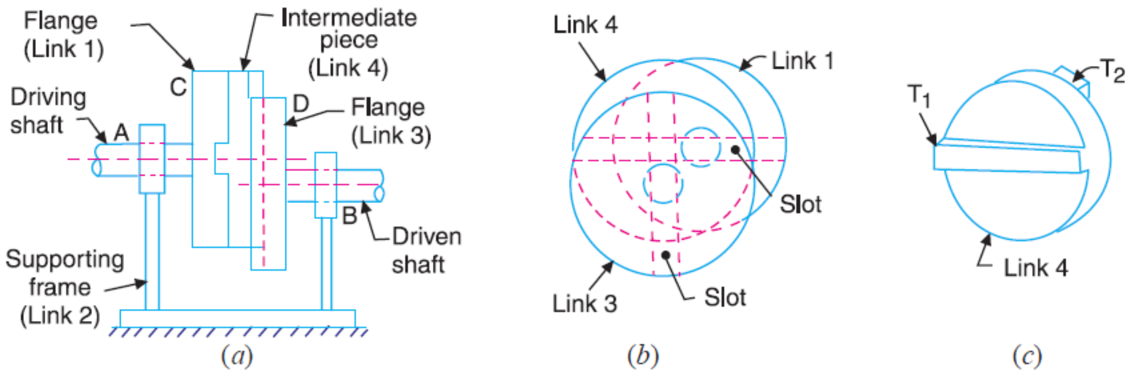


Figure. Oldham's coupling.

When the driving shaft *A* is rotated, the flange *C* (link 1) causes the intermediate piece (link 4) to rotate at the same angle through which the flange has rotated, and it further rotates the flange *D* (link 3) at the same angle and thus the shaft *B* rotates. Hence links 1, 3 and 4 have the same angular velocity at every instant. A little consideration will show, that there is a sliding motion between the link 4 and each of the other links 1 and 3. If the distance between the axes of the shafts is constant, the centre of intermediate piece will describe a circle of radius equal to the distance between the axes of the two shafts. Therefore, the maximum sliding speed of each tongue along its slot is equal to the peripheral velocity of the centre of the disc along its circular path.

Let ω = Angular velocity of each shaft in rad/s, and

r = Distance between the axes of the shafts in metres.

∴ Maximum sliding speed of each tongue (in m/s),

$$v = \omega . r$$