DEPARTMENT OF AEROSPACE ENGINEERING

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UNIT II - FLOW THROUGH CIRCULAR CONDUITS

Viscous flow

Viscous flow is the movement of a material in which the strength of the material is proportional to the strain rate. It can be characterized by the following:

- **Laminar flow**

  A smooth, parallel flow where layers of fluid glide over each other near solid boundaries.
- **Turbulent flow**

  A chaotic flow with eddies and intense mixing.
- **Viscosity**

  The internal frictional resistance to flow, which varies with the applied stress, strain rate, and temperature.
- **Poiseuille flow**

  Occurs when the pore radius is larger than the mean free path of gas molecules.
- **Sintering**

  Involves viscous flow and atomic diffusion of metallic glass powder in the supercooled liquid region.

Navier Stokes Equations – Definition

In fluid mechanics, the Navier-Stokes equations are partial differential equations that express the flow of viscous fluids. These equations are generalisations of the equations developed by Leonhard Euler (18th century) to explain the flow of frictionless and incompressible fluids. In 1821, Claude-Louis Navier put forward the component of viscosity (friction) for a more realistic and difficult problem of viscous fluids. During the entire middle period of the 19th century, George Gabriel Stokes refined this work even though entire solutions were found only in the case of basic two-dimensional flows. The complicated turbulence or vortices, or chaos that happens in three-dimensional fluid flows as velocities rise, has become intractable to any but numerical analysis techniques. The Navier–Stokes equations numerically describe the conservation of mass and the conservation of momentum for Newtonian fluids.

The Navier Stokes momentum equation

The Navier–Stokes momentum equation can be mathematically deduced as a distinct type of the Cauchy momentum equation. The general convective structure is

$$\frac{Du}{Dt} = \frac{1}{\rho} \nabla \cdot \sigma + g$$

by making the Cauchy stress tensor σ be the sum of a viscosity term τ (the deviatoric stress) and a pressure quantity -pI (volumetric stress), we arrive at,

Cauchy momentum equation (convective structure):

$$\rho \frac{Du}{Dt} = - \nabla p + \nabla \cdot \tau + \rho g$$

Where

- D/Dt is the material derivative, stated as
- $\frac{\partial}{\partial t} + u \cdot \nabla$
- ρ = density,
- u = flow velocity,

- ∇ = divergence,
- p = pressure,
- t = time,
- τ = deviatoric stress tensor (order 2),
- "g" denotes material accelerations acting on the continuum (like electrostatic accelerations, inertial acceleration, gravity, etc.)

Continuity Equation

The additional equation that represents the behaviour of fluid is the continuity equation. The equation to the conservation of mass implies the mass of the fluid is neither created nor destroyed in motion. The concept of conservation is an essential principle used throughout classical physics.

Continuity equation for flow density,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0$$

Cauchy momentum equation (conservation structure)

$$\rho \frac{Du}{Dt} = - \nabla p + \nabla \cdot \tau + \rho g$$

Every non-relativistic balance equation, such as the Navier–Stokes equations, can be constructed by starting with the Cauchy equations and citing the stress tensor with a constitutive relation. By describing the deviatoric stress tensor with fluid velocity gradient and viscosity, and taking fixed viscosity, the Cauchy equations will result in the Navier–Stokes equations.

Applications of Navier Stokes Equations

The Navier–Stokes equations can be very useful in applied physics. Primarily, they help to describe the mechanics of various engineering and scientific phenomena. They could be applied to model ocean currents, weather, air flow around wings, and the flow of water in pipes. These equations, in their simplified and full forms, help out with the modelling of vehicles and aircraft. They are also applied in the analysis of dense liquids, the examination of pollution, the design of power, and other processes related to fluids. Along with Maxwell's equations, these equations can be applied to study and model magnetohydrodynamics.

The Navier–Stokes equations also have great importance in pure mathematics. Despite their extensive range of applications, there is no proof for the consistent existence of smooth solutions in three dimensions; the equations are infinitely differentiable at every point in the domain. It is known as the Navier–Stokes smoothness and existence problem. This has been called one of the most significant unsolved problems in mathematics. A university has offered prize money of 1 million US dollars for whoever finds a solution for it.

Flow Velocity

Flow velocity is a vector field; to all points in a normal fluid, at any instance in a time period, it provides a vector whose magnitude and direction are of the fluid's velocity at that instance in time and at that point in space. It is generally examined in three dimensions. Even though two-dimensional and steady-state scenarios are usually employed as models, and greater dimensional analogues are analysed both in applied and pure mathematics, it is generally examined in three spatial dimensions. When the measurement of velocity field is done, other quantities such as temperature or pressure could be found utilising dynamical relations and equations. This is much different from what is commonly seen in early mechanics, where derived solutions are generally trajectories of a particle's position or deflection.

Solution of Navier-Stokes Equations

In the most general form, there are no analytical solutions to the Navier-Stokes equations. In other words, it is only possible to get some form of analytical solutions in particular approximate scenarios. The outcomes may not ever be realised in a real system. More geometrically sophisticated systems will need a numerical technique to find some form of a solution which is achieved with CFD simulations.