



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



DEPARTMENT OF MATHEMATICS

Properties of Group:

1] The identity element in a group is unique.

2] The inverse element in a group is unique

3] Cancellation law:

$$\forall a, b \in G \quad \text{i). } a * b = a * c \Rightarrow b = c \quad \text{[Left cancellation]}$$

$$\text{ii). } b * a = c * a \Rightarrow b = c \quad \text{[Right cancellation]}$$

4]. Let G be a group.

$$\text{If } a, b \in G \text{ then } (a * b)^{-1} = b^{-1} * a^{-1}$$

Proof:

Let $a, b \in G$

and a^{-1}, b^{-1} be their inverses resply.

Let $(G, *)$ be a group.

Let $a, b \in G$.

$$a * a^{-1} = a^{-1} * a = e$$

$$b * b^{-1} = b^{-1} * b = e$$

$$\text{Now } (a * b) * (b^{-1} * a^{-1}) = a * (b * b^{-1}) * a^{-1} \quad \text{(Associative)}$$

$$= (a * e) * a^{-1}$$

$$= a * a^{-1} \quad \text{Identity}$$

$$(a * b) * (b^{-1} * a^{-1}) = e \rightarrow (1)$$

$$\text{Similarly } (b^{-1} * a^{-1}) * (a * b) = e \rightarrow (2)$$

$$\text{From (1) and (2), } (a * b)^{-1} = b^{-1} * a^{-1}$$

The inverse of $(a * b)$ is $b^{-1} * a^{-1}$

Hence proved.

5]. Prove that a group $(G, *)$ is abelian iff

$$(a * b)^2 = a^2 * b^2, \quad \forall a, b \in G$$

Proof:

Assume that G is abelian

$$\therefore a * b = b * a, \quad a, b \in G \rightarrow (1)$$



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Now

$$\begin{aligned}
 a^2 * b^2 &= (a * a) * (b * b) \\
 &= a * [a * (b * b)] && \text{Associative} \\
 &= a * [(a * b) * b] && \text{Associative} \\
 &= a * [(b * a) * b] && \text{By (1)} \\
 &= (a * b) * (a * b) && \text{Associative} \\
 &= (a * b)^2
 \end{aligned}$$

conversely,
Assume that $(a * b)^2 = a^2 * b^2$

$$\begin{aligned}
 (a * b) * (a * b) &= (a * a) * (b * b) \\
 a * [b * (a * b)] &= a * [a * (b * b)] \\
 b * (a * b) &= a * (b * b) && \text{left cancellation law} \\
 (b * a) * b &= (a * b) * b \\
 b * a &= a * b && \text{Right cancellation law}
 \end{aligned}$$

$\Rightarrow G_1$ is abelian.

b). If $(G_1, *)$ is an abelian group, ST
 $(a * b)^n = a^n * b^n$, $\forall a, b \in G_1$, where n is a +ve integer
 proof:

since $(G_1, *)$ is abelian, we've

$$a * b = b * a, \forall a, b \in G_1 \rightarrow (1)$$

for $a, b \in G_1$, we've $(a * b)^1 = (b * a)^1$ by (1)

and $(a * b)^2 = (a * b) * (a * b)$

$$\begin{aligned}
 &= a * (b * a) * b && \text{Associative} \\
 &= a * (a * b) * b && \text{By (1)} \\
 &= (a * a) * (b * b) && \text{Associative} \\
 &= a^2 * b^2
 \end{aligned}$$

Thus, the result is true for $n=1, 2, \dots$
 let us assume that the result is valid for
 $n=m$.



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$$(a * b)^m = a^m * b^m \rightarrow (2)$$

now

$$(a * b)^{m+1} = (a * b)^m * (a * b)$$

$$= (a^m * b^m) * (a * b) \quad \text{By (2)}$$

$$= a^m * (b^m * a) * b \quad \text{Associative}$$

$$= a^m * (a * b^m) * b \quad (G \text{ is abelian)}$$

$$= (a^m * a) * (b^m * b)$$

$$(a * b)^{m+1} = a^{m+1} * b^{m+1}$$

Hence by induction, the result is true for +ve integral values of n .

i). In a group G , Prove that an elt. $a \in G$ such that $a^2 = e$, $a \neq e$ iff $a = a^{-1}$

Proof:

Assume that $a = a^{-1}$

To prove: $a^2 = e$

$$\begin{aligned} \text{Now consider } a^2 &= a * a \\ &= a * a^{-1} \\ &= e \end{aligned}$$

Conversely, Assume $a^2 = e$

To prove: $a = a^{-1}$

$$\text{Now } a^2 = e$$

$$a * a = e$$

$$a^{-1} * (a * a) = a^{-1} * e \quad [\text{Pre-multiply by } a^{-1}]$$

$$(a^{-1} * a) * a = a^{-1}$$

$$e * a = a^{-1}$$

$$a = a^{-1}$$

ii). If every elt. of a group G has its own inverse, then G is abelian.

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Proof:

Let $(G, *)$ be a group.

for $a, b \in G$, we've $a * b \in G$

Given $a = a^{-1}$ and $b = b^{-1}$

$$\text{Now } a * b = (a * b)^{-1}$$

$$= b^{-1} * a^{-1}$$

$$= b * a$$

(It has its own inverse)

(By property)

$\therefore G$ is abelian

Converse need not be true.