



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



DEPARTMENT OF MATHEMATICS

Duality in lattice :

When " \leq " is a partial order relation on a set S , then its converse " \geq " is also partial order relation on S .

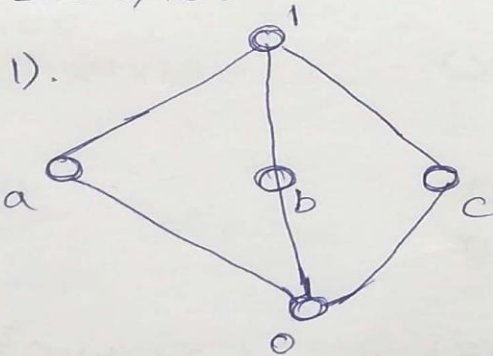
Distributive lattice :

A lattice (L, \wedge, \vee) is said to be distributive lattice if \wedge and \vee satisfies the following conditions, $\forall a, b, c \in L$.

$$D_1 : a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$

$$D_2 : a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$

Example :



$$L = \{0, a, b, c, 1\}$$

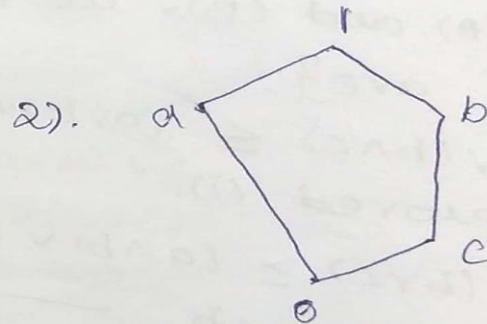
Here

$$a \vee (b \wedge c) = a \vee 0 = a$$

$$(a \vee b) \wedge (a \vee c) = 1 \wedge 1 = 1$$

since $a \neq 1$

It is not a distributive lattice



$$L = \{0, a, b, c, 1\}$$

Here

$$c \wedge (a \vee b) = c \wedge 1 = c$$

$$(c \wedge a) \vee (c \wedge b) = 0 \vee 0 = 0$$

$$c \wedge (a \vee b) \neq (c \wedge a) \vee (c \wedge b)$$

Not a distributive lattice.



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Theorem 3.1:

Prove that any chain is a distributive lattice.

Proof:

Let (L, \wedge, \vee) be a given chain and $\forall a, b \in L$.
Since any 2 elems. of a chain are comparable,
we've either $a \leq b$ or $b \leq a$.

Case 1: $a \leq b$	Case 2: $b \leq a$
Then $\text{GILB}\{a, b\} = a$	Then $\text{GILB}\{a, b\} = b$
$\text{LUB}\{a, b\} = b$	$\text{LUB}\{a, b\} = a$

In both cases, any 2 elems. of a chain has both GILB and LUB.

\therefore Any chain is a lattice.

Next we prove (L, \wedge, \vee) satisfies distributive property.

Let $a, b, c \in L$.
Since any chain satisfies comparable property,
we've the following 6 cases.

Case 1: $a \leq b \leq c$

2: $a \leq c \leq b$

3: $b \leq a \leq c$

4: $b \leq c \leq a$

5: $c \leq a \leq b$

6: $c \leq b \leq a$

Case 1: $a \leq b \leq c$

Prove $D_1: a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

LHS

$a \vee (b \wedge c)$

$\Rightarrow a \vee b$ ($\because b \leq c$)

$\Rightarrow b$ ($\because a \leq b$)

RHS

$(a \vee b) \wedge (a \vee c)$

$\Rightarrow b \wedge c$

$\Rightarrow b$

LHS = RHS



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$\therefore D_1$ condition is true for the case 1.
Similarly, we can easily prove D_1 -property to the remaining 5 cases.

$\therefore (L, \wedge, \vee)$ is a distributive lattice.

\therefore Any chain is a distributive lattice.

Theorem: 2 Modular Inequality

~~Let~~ If (L, \wedge, \vee) is a lattice, then for any $a, b, c \in L$, $a \leq c \Leftrightarrow a \vee (b \wedge c) \leq (a \vee b) \wedge c$

Proof:

Assume $a \leq c \rightarrow (1)$

$\therefore a \vee c = c$

By distributive inequality,

$$a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$$

$$\Rightarrow a \vee (b \wedge c) \leq (a \vee b) \wedge c \quad (\text{using (1)})$$

$$a \leq c \Rightarrow a \vee (b \wedge c) \leq (a \vee b) \wedge c \rightarrow (2)$$

Now conversely, assume

$$a \vee (b \wedge c) \leq (a \vee b) \wedge c$$

Now, by the defn. of LUB and GILB, we've

$$a \leq a \vee (b \wedge c) \leq (a \vee b) \wedge c \leq c$$

$$\Rightarrow a \leq c$$

$$\therefore a \vee (b \wedge c) \leq (a \vee b) \wedge c \Rightarrow a \leq c \rightarrow (3)$$

From (2) and (3), we've

$$a \leq c \Leftrightarrow a \vee (b \wedge c) \leq (a \vee b) \wedge c$$



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Problem

1. In any distributive lattice (L, \wedge, \vee) , $\forall a, b, c \in L$.
prove that $a \vee b = a \vee c$, $a \wedge b = a \wedge c \Rightarrow b = c$

Soln.:

$$b = b \vee (b \wedge a) \quad (\text{Absorption law})$$

$$= b \vee (a \wedge b)$$

$$= b \vee (a \wedge c) \quad \text{Givn. condition}$$

$$= (b \vee a) \wedge (b \vee c)$$

$$= (a \vee b) \wedge (b \vee c)$$

$$= (a \vee c) \wedge (b \vee c) \quad \text{Givn. condition}$$

$$= (a \wedge b) \vee c$$

$$= (a \wedge c) \vee c \quad \text{Givn. condition}$$

$$= c \wedge (c \wedge a) \vee c$$

$$b = c \quad \text{Absorption law}$$