NORMAL SUBGROUPS:

Let H be a subgroup of G under $*$.
Then H is said to be a normal subgroup of G, for every $x \in G$ and for $h \in H$,
if $x * h * x^{-1} \in H$
$x * H * x^{-1} \subseteq H$

Alternatively, a subgroup H of G is called a normal subgroup of G if $x * h = h * x$ for all $x \in G$.

PROPERTIES:

1) A subgroup H of a group G is normal if $x * h * x^{-1} = H$ for all $x \in G$.
2) Every subgroup of an abelian group is normal.
3) If G is a group of prime order, then G has no proper subgroups.
4) Let $(G, *)$ be a group. Let $H = \{a / a \in G$ and $a * b = b * a$, $\forall\ b \in G\}$. Show that H is a normal subgroup.
5) The intersection of any two normal subgroups of a group is a normal subgroup.

proof: Let H and K are normal subgroups.
$\Rightarrow$ H and K are subgroups of G.
$\Rightarrow$ $H \cap K$ is a subgroup of G.

To prove: $H \cap K$ is normal.

Let $x \in G$ and $h \in H \cap K$

$x \in G$, $h \in H$ and $h \in K$

$\Rightarrow x \in G$ & $h \in H \Rightarrow x * h * x^{-1} \in H$. —— (1)

& also $x \in G$ & $h \in K \Rightarrow x * h * x^{-1} \in K$. —— (2)

Since H and K are normal subgroups.

from (1) & (2), we get,

$x * h * x^{-1} \in H \cap K$

$\Rightarrow H \cap K$ is a normal subgroup of G.