SNS College of Technology.

Coimbatore - 35

Internal I - Answer Key - Set A.

19MAT301 - Discrete Mathematics

Part A.

1.

P	Q	P∧Q	Q∨P
T	T	T	T
F	T	F	T
T	F	F	T
F	F	T	F

2. If the crops will not grow then it does not rains. $(\neg Q \rightarrow \neg P)$

3. $\forall x \left[A(x) \rightarrow R(x) \right]$

4. Computer Science → networking to design routing protocols
   Algorithms → optimizing result

5. The principle of mathematical is a technique for providing that a statement is true for all natural numbers.

Part B

6) a) i)

P∧Q	¬(P∨Q)	P∧Q ∧ ¬(P∨Q)
T	F	F
F	F	F
F	F	F
F	T	F

ii)

Step		Premises	Rule	
	1	R	P (Assumed)	
	2	$\neg R \vee P$	P	
{1,2}	3	P	T	$\neg P, P \vee Q \Rightarrow Q$
	4	$P \to (Q \to S)$	P	
{3,4}	5	$Q \to S$	T	$P, P \to Q \Rightarrow Q$
	6	Q	P	
{5,6}	7	S	T	$P, P \to Q \Rightarrow Q$
{1,7}	8	$R \to S$	CP	

b) $\neg(P \wedge Q) \to (\neg P \vee (\neg P \vee Q))$

i)

$\Leftrightarrow \neg(\neg(P \wedge Q)) \vee (\neg P \vee (\neg P \vee Q))$ M.I

$\Leftrightarrow (P \wedge Q) \vee (\neg P \vee (\neg P \vee Q))$ Involution (DN law)

$\Leftrightarrow (P \wedge Q) \vee ((\neg P \vee \neg P) \vee Q)$ Ass

$\Leftrightarrow (P \wedge Q) \vee (\neg P \vee Q)$ Idem

$\Leftrightarrow P \wedge Q \vee (\neg P \vee Q)$ Distri

$\Leftrightarrow (P \vee (\neg P \vee Q)) \wedge (Q \vee (\neg P \vee Q))$ Asso Comm

$\Leftrightarrow ((P \vee \neg P) \vee Q) \wedge (Q \vee (Q \vee \neg P))$ comple Asso

$\Leftrightarrow (T \vee Q) \wedge ((Q \vee Q) \vee \neg P)$ Comm, Idem

$\Leftrightarrow (Q \vee T) \wedge (Q \vee \neg P)$ Comm, Dominance

$\Leftrightarrow T \wedge (\neg P \vee Q)$ Comm law

$\Leftrightarrow (\neg P \vee Q) \wedge T$ Id law.

ii) $\Leftrightarrow \neg P \vee ((P \to Q) \wedge \neg(\neg Q \vee \neg P))$

$\Leftrightarrow \neg P \vee ((\neg P \vee Q) \wedge \neg(\neg Q \vee \neg P))$

$\Leftrightarrow \neg P \vee [(\neg P \vee Q) \wedge (Q \wedge P)]$

$\Leftrightarrow \neg P \vee [(\neg P \wedge P) \wedge Q \vee (Q \wedge Q) \wedge P]$

$\Leftrightarrow \neg P \vee [F \wedge Q) \vee (Q \wedge P)]$

$\Leftrightarrow \neg P [F \vee (P \wedge Q)]$

$\Leftrightarrow (\neg P \wedge T) \vee (P \wedge Q)$

$\Leftrightarrow (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (P \wedge Q)$

7) a) i)

Step	premis	Rule
1	P	neg of Conc
2	P→Q	P
3	Q	T
4	R→¬Q	P
5	¬R	T
6	RVS	P
7	S	T
8	S→¬Q	P
9	¬Q	T
10	Q∧¬Q	T
11	F	T

7) a ii)

Step	Premises	Rule
1	P→Q	P
2	Q→R	P
3	P→R	T
4	S→¬R	P
5	R→¬S	T
6	P→¬S	T
7	¬P∨¬S	T
8	¬(P∧S)	T
9	P∧S	P
10	(P∧S)∧¬(P∧S)	T
11	F	T

7) b) i) $P(1): \frac{1}{2} = 1/2$, $P(k) = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots \frac{1}{k(k+1)} = \frac{k}{k+1}$

$P(k+1) = \frac{k}{(k+1)} + \frac{1}{(k+1)(k+2)} = \frac{k+1}{(k+1)+1}$ is true

ii) $P(1) \Rightarrow 5 = 5$, $P(k) = 8^k - 3^k = 5m$

$P(k+1) = 8^{k+1} - 3^{k+1} = (3^k + 5m)\cdot 8 - 3^k - 3$

$= 5(8m + 3^k)$ is x of 5, ∀n

8) a) $\exists x [A(x) \wedge J(x)]$, $\forall x [J(x) \rightarrow H(x)]$

Conclusion: $\exists x [A(x) \wedge H(x)]$

Step	premises	Rule
1	$\exists x (A(x) \wedge J(x))$	P
2	$A(y) \wedge J(y)$	ES
3	$A(y)$	T
4	$J(y)$	T
5	$\forall x [J(x) \rightarrow H(x)]$	P
6	$J(y) \rightarrow H(y)$	US
7	$H(y)$	T
8	$A(y) \wedge H(y)$	T
9	$\exists x [A(x) \wedge H(x)]$	EG

8) b) i) $P(1)$: $1 = 1$

$$P(k) = \frac{k(k+1)(2k+1)}{6}$$

$$P(k+1) = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{(k+1)(k+2)(2k+3)}{6} = \frac{(k+1)[(k+1)+1][2(k+1)+1]}{6}$$

is true $\forall n$

8 (bi) Pigeon hole principle

If $(n+1)$ pigeon occupies n holes then atleast one hole has more than one pigeon.

(ii) No. of Pigeons m = 40,325

No. of holes n = 25

$$\left[\frac{m+1}{1}\right] + 1 = 1614 \text{ pages.}$$