UNIT - III

Analysis of continuous Time Signals

Fourier Series :- Fourier series is used to analyse the periodic signals. The harmonic content of the signals are analysed with the help of fourier series. Fourier series can be developed for continuous time (CT) and Discrete time (DT).

Types of Fourier series :-

(i) Trignometric Fourier series (or) Quadrature Fourier series
(ii) compact trignometric Fourier series (or) polar Fourier series
(iii) Exponential Fourier series.

Trignometric Fourier series :-

$$x(t) = a(0) + \sum_{k=1}^{\infty} a(k) \cos k\omega_0 t + \sum_{k=1}^{\infty} b(k) \sin k\omega_0 t$$

$$a(0) = \frac{1}{T} \int_{\langle T \rangle} x(t)\, dt$$

$$a(k) = \frac{2}{T} \int_{\langle T \rangle} x(t) \cos k\omega_0 t \, dt$$

$$b(k) = 2/T \int_{\langle T \rangle} x(t) \sin k\omega_0 t \, dt$$

$$\boxed{\omega_0 = 2\pi/T}$$ $T \rightarrow$ period of signal $x(t)$

compact Trignometric Fourier series :-

$$x(t) = D(0) + \sum_{k=1}^{\infty} D(k) \cos\left(k\omega_0 t + \phi_k\right)$$

$$D(0) = a_0 = \frac{1}{T} \int_{\langle T \rangle} x(t)\, dt$$

$$D(k) = \sqrt{a(k)^2 + b(k)^2}$$

$$\phi(k) = -\tan^{-1}\left[\frac{b(k)}{a(k)}\right]$$

Exponential Fourier series :-

$$x(t) = \sum_{k=-\infty}^{\infty} X(k)\, e^{jk\omega_0 t} \rightarrow \text{synthesis equation}$$

$$X(k) = \frac{1}{T} \int_{\langle T \rangle} x(t) \cdot e^{-jk\omega_0 t}\, dt \rightarrow \text{Analysis equation}$$

$x(t)$ & $X(k)$ forms Fourier transform pair

Convergence of Fourier series (or) (Dirichlet's conditions) :-

(i) Single value property :-

$x(t)$ must have only one value at any time constant with in the interval T_0.

(ii) Finite discontinuits :-

$x(t)$ should have atmost finite no of discontinuits in the interval T_0.

(iii) Finite peaks :-

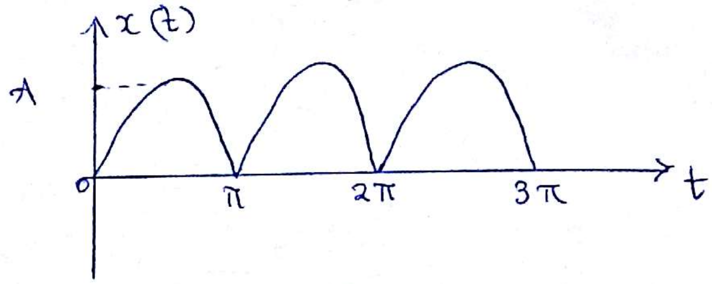
The signal $x(t)$ must have finite no of maxima and minima in the interval T_0.

(iv) Absolute Integrability :-

The signal $x(t)$ should be absolutely integrable

$$\int_{\langle T_0 \rangle} |x(t)|\, dt < \infty$$

Find the trignometric fourier Series for full wave rectifier output.



$$x(t) = \begin{cases} A \sin t, & 0 \le t \le \pi \\ 0 & \text{otherwise} \end{cases}$$

Time period $= T_0 = T = \pi$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2$$

$$a(0) = \frac{1}{T}\int_{<T>} x(t)\, dt$$

$$= \frac{1}{\pi}\int_0^{\pi} A \sin t\, dt$$

$$= A/\pi \left[\frac{-\cos t}{1}\right]_0^{\pi}$$

$$= A/\pi \left[-\cos\pi + \cos 0\right]$$

$$= \frac{A}{\pi}\left[-(-1)+1\right] \Rightarrow A/\pi \left[1+1\right]$$

$$a(0) = \frac{2A}{\pi}$$

$$a(k) = 2/T \int_T x(t) \cos k\omega_0 t\, dt$$

$$= 2/\pi \int_0^{\pi} A \sin t \cos k\, 2t\, dt$$

$$\sin A \cos B = \tfrac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$= \frac{\cancel{2}A}{\cancel{2}\pi}\int_0^{\pi}\left[\sin\left(t+k2t\right) + \sin\left(t-k2t\right)\right] dt$$

$$= A/\pi \int_0^{\pi}\left[\sin(1+2k)t + \sin(1-2k)t\right] dt$$

$$= \frac{A}{\pi}\left[\frac{-\cos(1+2k)t}{1+2k} - \frac{\cos(1-2k)t}{1-2k}\right]_0^{\pi}$$

$$= \frac{A}{\pi}\left\{\left[\frac{-\cos\pi(1+2k)}{1+2k} - \frac{\cos\pi(1-2k)}{1-2k}\right] - \left[\frac{-\cos 0}{1+2k} - \frac{\cos 0}{1-2k}\right]\right\}$$

$$= \frac{A}{\pi}\left[\frac{-\cos\pi(1+2k)}{1+2k} - \frac{\cos\pi(1-2k)}{1-2k} + \frac{1}{1+2k} + \frac{1}{1-2k}\right]$$

$$\left[\because \cos A \mp B = \cos A \cos B \mp \sin A \sin B\right]$$

$$= \frac{A}{\pi}\left\{\left[\frac{-(\cos\pi\cos 2k\pi + \sin\pi \sin 2k\pi)}{1+2k} + \frac{1}{1+2k}\right] + \left[\frac{-(\cos\pi\cos 2k\pi - \sin\pi \sin 2k\pi)}{1-2k} + \frac{1}{1-2k}\right]\right\}$$

$$= \frac{A}{\pi}\left[\frac{-(-1)}{1+2k} + \frac{1}{1+2k} - \frac{(-1)}{1-2k} + \frac{1}{1-2k}\right]$$

$$= \frac{A}{\pi}\left[\frac{2}{1+2k} + \frac{2}{1-2k}\right] \Rightarrow \frac{A}{\pi}\left[\frac{4}{1-4k^2}\right]$$

$$a(k) = \frac{4A}{\pi(1-4k^2)}$$

$$b(k) = \frac{2}{T}\int_{\langle T\rangle} x(t)\sin k\omega_0 t\, dt$$

$$\because \sin A \sin B = \frac{1}{2}\left[\cos(A+B) - \cos(A-B)\right]$$

$$= \frac{2}{\pi}\int_0^{\pi} A \sin t \sin k2t\, dt$$

$$= \frac{2A}{2\pi}\int_0^{\pi}\left[\cos(t+2kt) - \cos(t-2kt)\right] dt$$

$$= \frac{A}{\pi}\int_0^{\pi}\left[\cos t(1+2k) - \cos t(1-2k)\right] dt$$

$$= \frac{A}{\pi}\left[\frac{\sin t(1+2k)}{1+2k} - \frac{\sin t(1-2k)}{1-2k}\right]_0^{\pi}$$

$$= \frac{A}{\pi}\left[0 - 0 - 0 + 0\right]$$

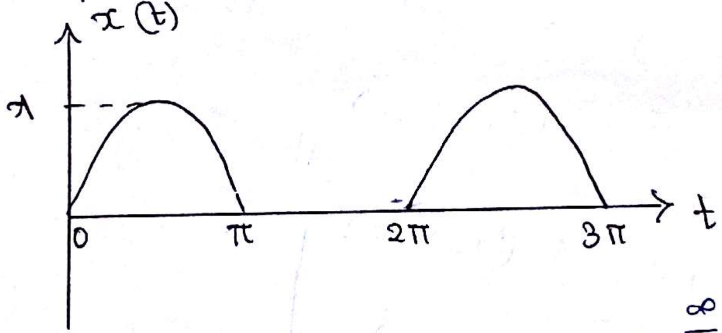
$$b(k) = \frac{A}{\pi}(0) \Rightarrow 0$$

$$x(t) = a_0 + \sum_{k=1}^{\infty} a(k) \cos k\omega_0 t + \sum_{k=1}^{\infty} b(k) \sin k\omega_0 t$$

$$= \frac{2A}{\pi} + \sum_{k=1}^{\infty} \frac{4A}{\pi(1-4k^2)} \cos k\omega_0 t + \sum_{k=1}^{\infty} (0) \sin k\omega_0 t$$

$$x(t) = \frac{2A}{\pi} + \sum_{k=1}^{\infty} \frac{4A}{\pi(1-4k^2)} \cos k\omega_0 t$$

2) Find the trignometric Fourier series of the half wave rectifier output.



$$x(t) = a(0) + \sum_{k=1}^{\infty} a(k) \cos k\omega_0 t + \sum_{k=1}^{\infty} b(k) \sin k\omega_0 t$$

$$x(t) = \begin{cases} A \sin t, & 0 \le t \le \pi \\ 0, & \pi \le t \le 2\pi \end{cases}$$

Time period $T_0 = T = 2\pi$

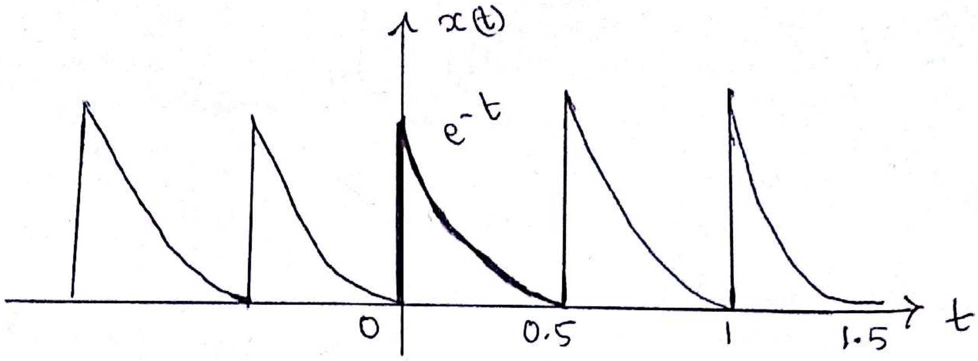
$$\omega_0 = \frac{2\pi}{T} = \frac{\cancel{2\pi}}{\cancel{2\pi}} = 1$$

$$a(0) = \frac{A}{\pi}, \quad a(k) = \frac{2A}{\pi(1-k^2)} \text{ for } k = 0, 2, 4 \ldots$$

$$b(k) = 0$$

$$\therefore x(t) = \frac{A}{\pi} + \sum_{k=0,2,4}^{\infty} \frac{2A}{\pi(1-k^2)} \cos kt$$

Find the exponential Fourier series :



$T = 0.5$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{0.5} = 4\pi$$

$$x(t) = \sum_{k=-\infty}^{\infty} X(k)\, e^{jk\omega_0 t}$$

$$X(k) = \frac{1}{T}\int_{\langle T \rangle} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{0.5}\int_0^{0.5} e^{-t}\, e^{-jk4\pi t}\, dt$$

$$= 2\int_0^{0.5} e^{-t(1+jk4\pi)}\, dt$$

$$= 2\left[\frac{e^{-t(1+jk4\pi)}}{-(1+jk4\pi)}\right]_0^{0.5}$$

$$= \frac{2\left[e^{-0.5}\, e^{-0.5j4\pi k} + 1\right]}{-(1+j4\pi k)}$$

$$= \frac{2\left[e^{-0.5}+1\right]}{-(1+j4\pi k)} \Rightarrow \frac{2\left(-e^{-0.5}+1\right)}{1+j4\pi k}$$

$$X(k) = \frac{2(-0.6065+1)}{1+j4\pi k} \Rightarrow \frac{0.787}{1+j4\pi k}$$

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{0.787}{1+j4\pi k}\, e^{jk\omega_0 t}$$