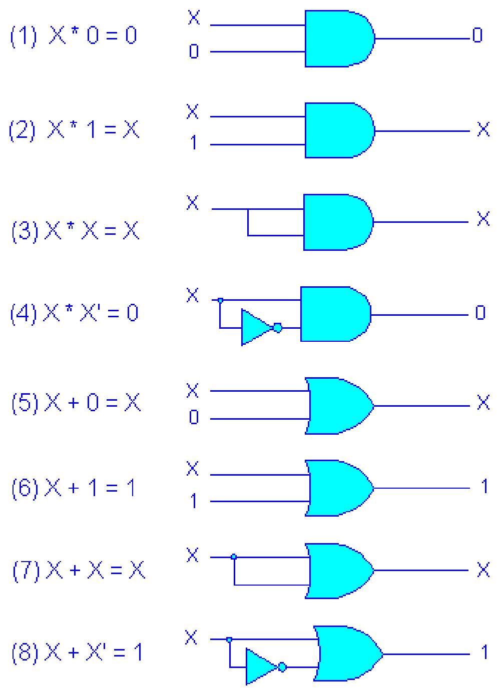
**Boolean Theorems**

Investigating the various Boolean theorems (rules) can help us to simplify logic expressions and logic circuits.




**Multivariable Theorems**

The theorems presented below involve more than one variable:

(9) x + y = y + x *(commutative law)*
(10) x * y = y * x *(commutative law)*
(11) x+ (y+z) = (x+y) +z = x+y+z *(associative law)*
(12) x (yz) = (xy) z = xyz *(associative law)*
(13a) x (y+z) = xy + xz
(13b) (w+x)(y+z) = wy + xy + wz + xz
(14) x + xy = x [proof see below]
(15) x + x'y = x + y

Proof of (14)

x + xy = x (1+y)

= x * 1 *[using theorem (6)]*

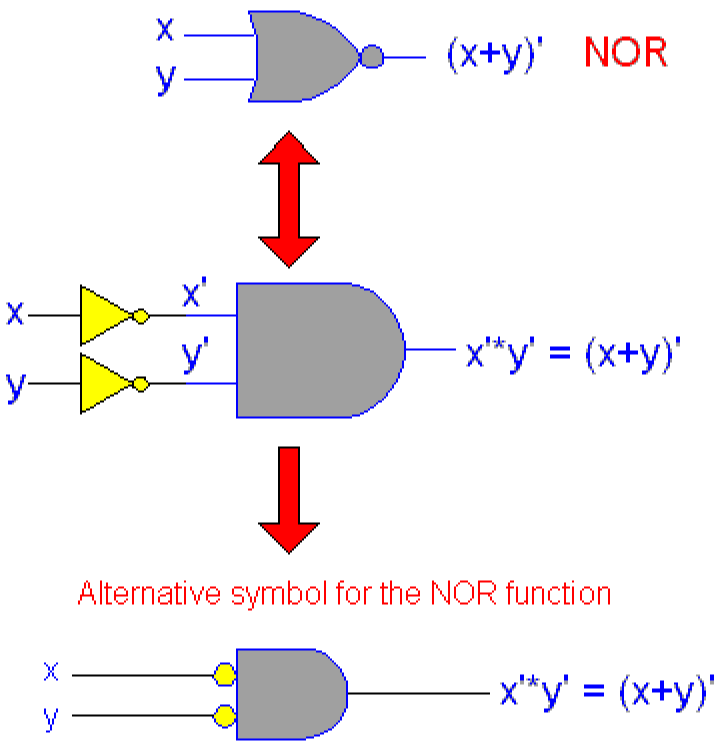
= x *[using theorem (2)]*

**DeMorgan's Theorem**

DeMorgan's theorems are extremely useful in simplifying expressions in which a product or sum of variables is inverted. The two theorems are:

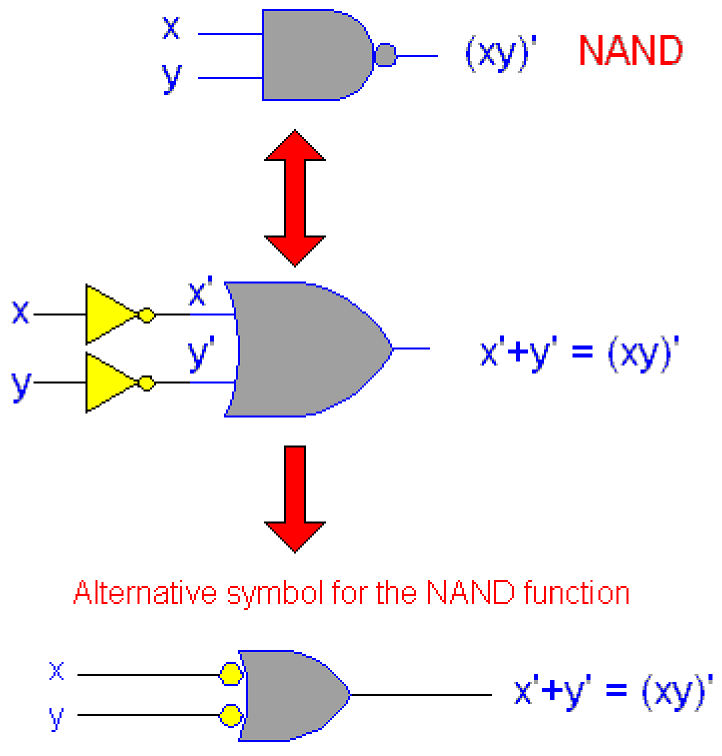
(16) (x+y)' = x' * y'

Theorem (16) says that when the OR sum of two variables is inverted, this is the same as inverting each variable individually and then ANDing these inverted variables.




(17) (x*y)' = x' + y'

Theorem (17) says that when the AND product of two variables is inverted, this is the same as inverting each variable individually and then ORing them.




**Minterm**

In a Boolean function, a binary variable (x) may appear either in its normal form (x) or in its complement form (x').Consider 2 binary variables x and y and an AND operation, there are 4 and only 4 possible combinations: x'•y', x'•y, x•y' & x•y
Each of the 4 product terms is called a MINTERM or STANDARD PRODUCT

By definition, a Minterm is a product which consists of all the variables in the normal form or the complement form but NOT BOTH.

e.g. for a function with 2 variables x and y:
x•y' is a minterm but x' is NOT a minterm
e.g. for a function with 3 variables x, y andz:
x'yz' is a minterm but xy' is NOT a minterm

**Maxterm**

Consider 2 binary variables x and y and an OR operation, there are 4 and only 4 possible combinations: x'+y', x'+y, x+y', x+y.Each of the 4 sum terms is called a MAXTERM or STANDARD SUM.By definition, a Maxterm is a sum in which each variable appears once and only once either in its normal form or its complement form but NOT BOTH.

**Minterms and Maxterms for 3 Variables**

| Variable | | | Minterm | | Maxterm | |
|---|---|---|---|---|---|---|
| x | y | z | Term | Designation | Term | Designation |
| 0 | 0 | 0 | x’y’z’ | $m_0$ | x+y+z | $M_0$ |
| 0 | 0 | 1 | x’y’z | $m_1$ | x+y+z’ | $M_1$ |
| 0 | 1 | 0 | x’yz’ | $m_2$ | x+y’+z | $M_2$ |
| 0 | 1 | 1 | x’yz | $m_3$ | x+y’+z’ | $M_3$ |
| 1 | 0 | 0 | xy’z’ | $m_4$ | x’+y+z | $M_4$ |
| 1 | 0 | 1 | xy’z | $m_5$ | x’+y+z’ | $M_5$ |
| 1 | 1 | 0 | xyz’ | $m_6$ | x’+y’+z | $M_6$ |
| 1 | 1 | 1 | xyz | $m_7$ | x’+y’+z’ | $M_7$ |

**Minterm Boolean Expression**

Boolean functions can be expressed with minterms,

e.g. $f1(x,y,z) = m1 + m4 + m6 = \Sigma m(1, 4, 6)$

$f2(x,y,z) = m2 + m4 + m6 + m7$

$= \Sigma m(2, 4, 6, 7)$

| x | y | z | f1 | f2 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 |

**Maxterm Boolean Expression**

Boolean functions can also be expressed with maxterms,

e.g. $f1' = x'y'z'+x'yz'+x'yz+xy'z+xyz$

$f1 = (x'y'z'+x'yz'+x'yz+xy'z+xyz)'$

$= (x+y+z)(x+y'+z)(x+y'+z')(x'+y+z')(x'+y'+z')$

$= M0 \bullet M2 \bullet M3 \bullet M5 \bullet M7$

$= \Pi M(0, 2, 3, 5, 7)$

$f2 = M0 \bullet M1 \bullet M3 \bullet M5$

$= \Pi M(0, 1, 3, 5)$

| x | y | z | f1 | f2 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 |

**Literal**

A Literal is a variable in a product or sum term

xy' is a 2-literal product term

x'yz has 3 literals

x' + xy' + x'yz is an expression of sum of products with 3 product terms.The 3 product terms have 1, 2 and 3 literals respectively

x'(x+y')(x'+y+z) is an expression of product of sums.The 3 sum terms have 1, 2 and 3 literals

**Express Boolean Functions in Minterms**

If product terms in a Boolean function are not minterms, they can be converted to minterms

e.g. f(a,b,c) = a' + bc' + ab'c

Function f has 3 variables, therefore, each minterm must have 3 literals

Neither a' nor bc' are minterms.They can be converted to minterm.ab'c is a minterm

**Conversion to Minterms**

e.g. f(a,b,c) = a' + bc' + ab'c

To convert a' to a minterm, the 2 variables (b, c) must be added, without changing its functionality .Since a'=a'•1 & 1 = b+b', a'= a'(b + b') = a'b + a'b'

Similarly, a'b = a'b(c + c') = a'bc + a'bc' and a'b' = a'b'(c+c') = a'b'c + a'b'c'

bc' = bc'(a+a') = abc' + a'bc'

f = a'bc+a'bc'+a'b'c+a'b'c'+abc'+a'bc'+ab'c

**Express Boolean Functions in Maxterms**

By using the Distribution Law: x+yz = (x+y)(x+z), a Boolean function can be converted to an expression in product of maxterms

e.g. f(a,b,c) = a'+bc'

= (a'+b)(a'+c') {not maxterms}

= (a'+b+cc')(a'+c'+bb') {cc'=0}

= (a'+b+c)(a'+b+c')(a'+c'+b)(a'+c'+b')

= (a'+b+c)(a'+b+c')(a'+c'+b')