Test for Difference of proportions

Null Hypothesis $H_0 : P_1 = P_2$

Test statistics : $$Z = \frac{P_1' - P_2'}{\sqrt{PQ \times \left[\frac{1}{n_1} + \frac{1}{n_2}\right]}}$$

where $P_1' = \frac{x_1}{n_1}$; $P_2' = \frac{x_2}{n_2}$

$$P = \frac{x_1 + x_2}{n_1 + n_2} \quad \text{and}$$

$$Q = 1 - P$$

1) Random samples of 400 men and 600 women were asked whether they would have a flavour near their residence 200 men and 325 women were in favour of the proportion. Test the hypothesis that proportion of men and women in favour of the proportion same against that they are not at 5% level.

Solution :

$n_1 = 400 \quad n_2 = 600$

$x_1 = 200 \quad x_2 = 325$

$$P_1' = \frac{x_1}{n_1} = \frac{200}{400} = 0.5$$

$$P_2' = \frac{x_2}{n_2} = \frac{325}{600} = 0.54$$

$$P = \frac{x_1 + x_2}{n_1 + n_2} = \frac{200 + 325}{400 + 60} = \frac{525}{1000} = 0.525$$

$q = 1 - p = 1 - 0.525 = 0.475$

Step 1 : Formulate H_0 and H_1

$H_0 : P_1 = P_2$

$H_1 : P_1 \neq P_2$ [Two tail]

Step 2 : Level of significance

$\alpha = 0.05$

Step 3 : Test Statistics

$$Z = \frac{P_1' - P_2'}{\sqrt{pq\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}}$$

$$= \frac{0.5 - 0.54}{\sqrt{(0.525)(0.475) \times \left[\frac{1}{400} + \frac{1}{600}\right]}}$$

$$= \frac{-0.04}{\sqrt{0.249\left[0.0025 + 0.0016\right]}}$$

$$= \frac{-0.04}{\sqrt{0.249\,(0.0041)}} = \frac{-0.04}{\sqrt{0.001}} = \frac{-0.04}{0.031}$$

$$= 1.29$$

$$\boxed{Z = 1.29}$$

Step 4 : Critical value

$Z_\alpha = 1.96$

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Step 5 : conclusion

$|Z| = 1.29 < 1.96 = |Z_\alpha|$

$\therefore H_0$ is accepted