SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
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DEPARTMENT OF BIOMEDICAL ENGINEERING

19BMB302 - BIOMEDICAL SIGNAL PROCESSING

III YEAR/ V SEMESTER

UNIT III INFINITE IMPULSE RESPONSE FILTERS

UNIT II INFINITE IMPULSE RESPONSE FILTERS

Characteristics of practical frequency selective filters.
Characteristics of commonly used analog filters
Butterworth filters, Chebyshev filters.
Design of IIR filters from analog filters (LPF, HPF, BPF, BRF)
Approximation of derivatives
Impulse invariance method
Bilinear transformation
Frequency transformation in the analog domain
Structure of IIR filter - direct form I, direct form II
Cascade, parallel realizations

Introduction

- Frequency-selective filters pass only certain frequencies
- Any discrete-time system that modifies certain frequencies is called a filter.
- We concentrate on design of causal Frequency-selective filters

- Filter : A input-selective device that allows only those inputs having some specified attribute passing through it.
- Frequency-selective filter => attribute is the frequency

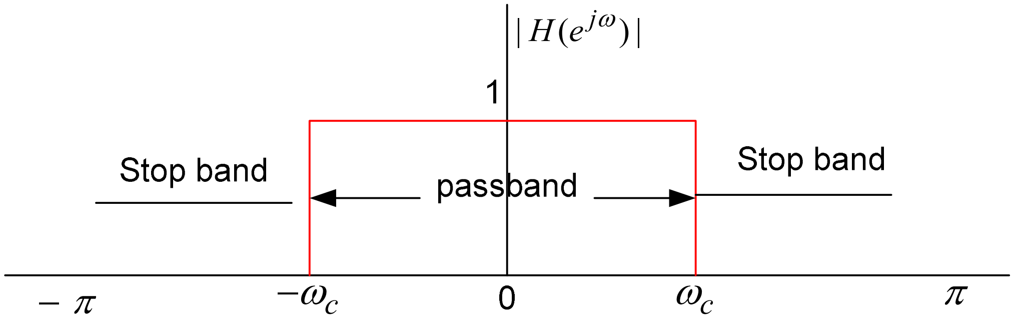
x(t) LTI Filter y(t)

$$Y(j\Omega) = H(j\Omega)X(j\Omega)$$

A weighting function to the different frequency component in x(t)

Ideal Filter Characteristics

Lowpass



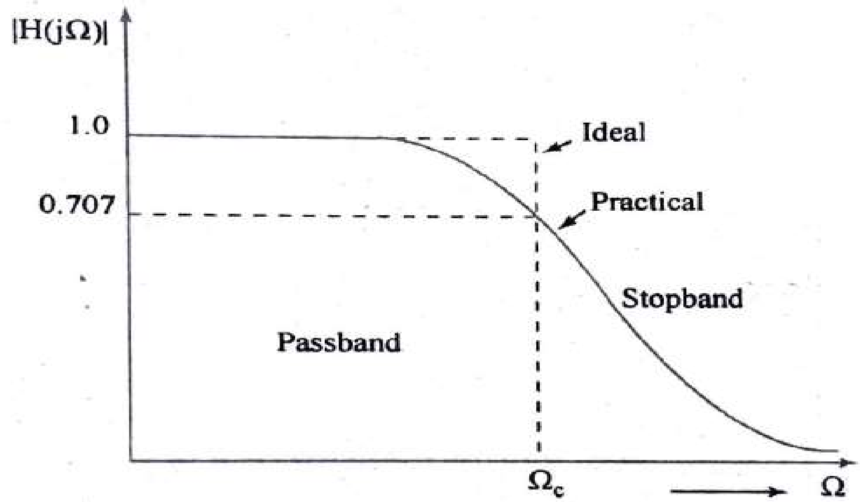
$$\omega_c = \text{cutoff frequency}$$

$$H(e^{j\omega}) = \begin{cases} 1 & |\omega| \le \omega_c \\ 0 & \omega_c < |\omega| \le \pi \end{cases}$$

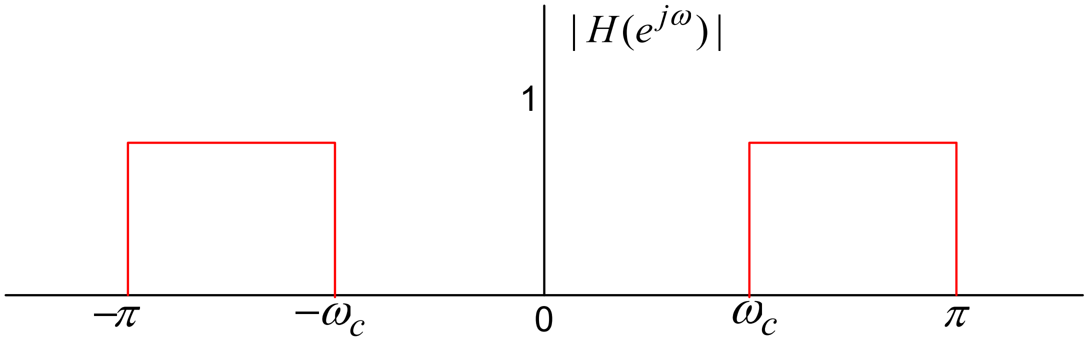
Low Pass filter

The magnitude response of an ideal lowpass filter allows low frequencies in the pass-band $0 < \Omega < \Omega_c$ to pass, whereas the higher frequencies in the stopband $\Omega > \Omega_c$ are blocked. The frequency Ω_c between the two bands is the cutoff frequency, where the magnitude $|H(j\Omega)| = 1/\sqrt{2}$.

In practice it is impossible to obtain the ideal response. The practical response of a lowpass filter is shown in solid line in Fig.



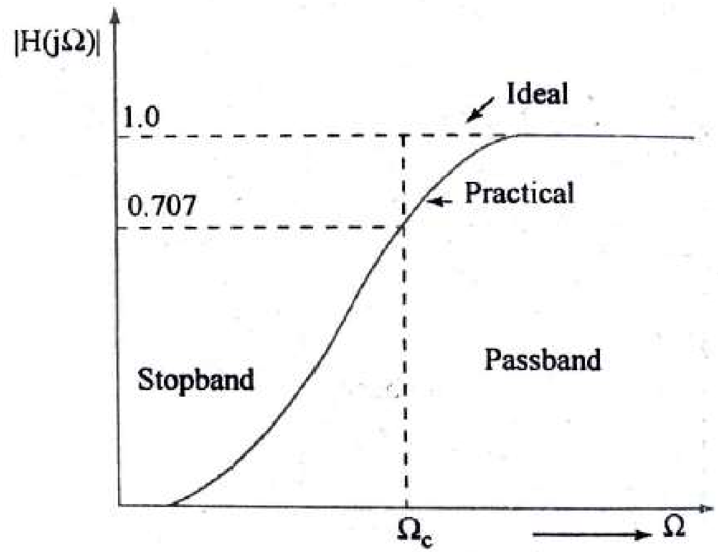
Highpass



$$H(e^{j\omega}) = \begin{cases} 1 & \omega_c < |\omega| \le \pi \\ 0 & |\omega| \le \omega_c \end{cases}$$

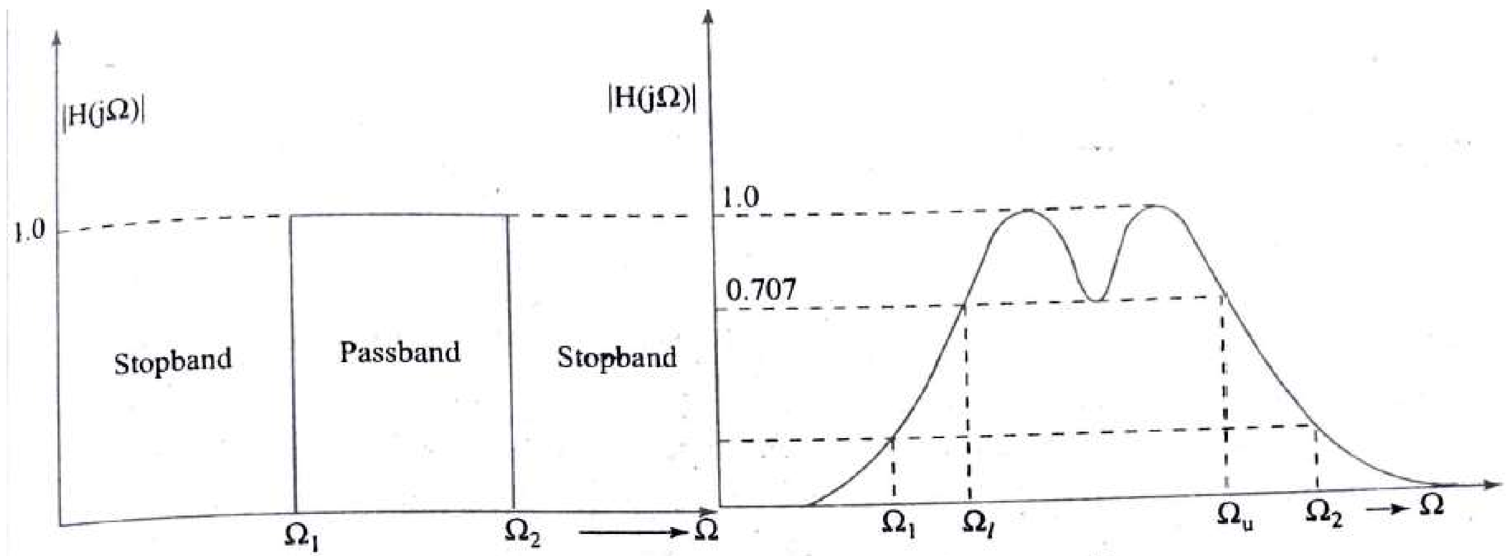
High Pass filter

The highpass filter allows high frequencies above $\Omega > \Omega_c$ and rejects the frequencies between $\Omega = 0$ and $\Omega = \Omega_c$. The magnitude response of an ideal and practical highpass filter is shown in Fig.



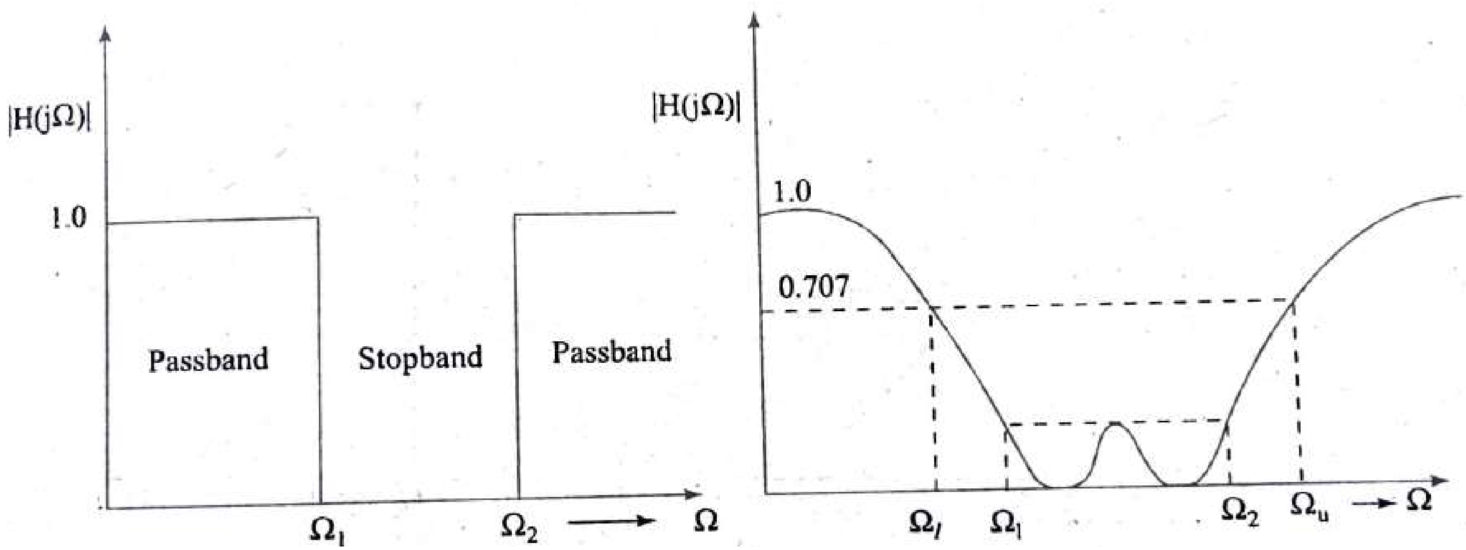
Band Pass filter

It allows only a band of frequencies Ω_1 to Ω_2 to pass and stops all other frequencies. The ideal and practical response of bandpass filter are shown in Fig.



Band stop filter

It rejects all the frequencies between Ω_1 and Ω_2 and allows remaining frequencies. The magnitude response of an ideal and practical filters is shown in Fig.



Design of Digital filters from Analog filters

The most common technique used for designing IIR digital filters known as indirect method, involves first designing an analog prototype filter and then transforming the prototype to a digital filter. For the given specifications of a digital filter, the derivation of the digital filter transfer function requires three steps.

1. Map the desired digital filter specifications into those for an equivalent analog filter.
2. Derive the analog transfer function for the analog prototype.
3. Transform the transfer function of the analog prototype into an equivalent digital filter transfer function.

Analog Filter Vs Digital Filter

Analog Filter	Digital Filter
1. Analog filter processes analog inputs and generates analog outputs.	1. A digital filter processes and generates digital data.
2. Analog filters are constructed from active or passive electronic components.	2. A digital filter consists of elements like adder, multiplier and delay unit.
3. Analog filter is described by a differential equation.	3. Digital filter is described by a difference equation.
4. The frequency response of an analog filter can be modified by changing the components.	4. The frequency response can be changed by changing the filter coefficients.

Advantages and disadvantages of digital filters

Advantages

1. Unlike analog filter, the digital filter performance is not influenced by component ageing, temperature and power supply variations.
2. A digital filter is highly immune to noise and possesses considerable parameter stability.
3. Digital filters afford a wide variety of shapes for the amplitude and phase responses.
4. There are no problems of input or output impedance matching with digital filters.
5. Digital filters can be operated over a wide range of frequencies.
6. The coefficients of digital filter can be programmed and altered any time to obtain the desired characteristics.
7. Multiple filtering is possible only in digital filter.

Disadvantage

1. The quantization error arises due to finite word length in the representation of signals and parameters.

IIR Vs FIR

	FIR	IIR
Impulse Response	finite	infinite
System Function	H(z)=P(z)	H(z)=P(z)/D(z)
Structure diagram	No feedback	Have feedback
Phase response	Exact linear phase	Not Necessarily a Linear-Phase
Zero-poles	Only have zeros	Both zeros and poles

Continuous-time IIR filters

- Butterworth filters
- Chebyshev Type I filters
- Chebyshev Type II filters

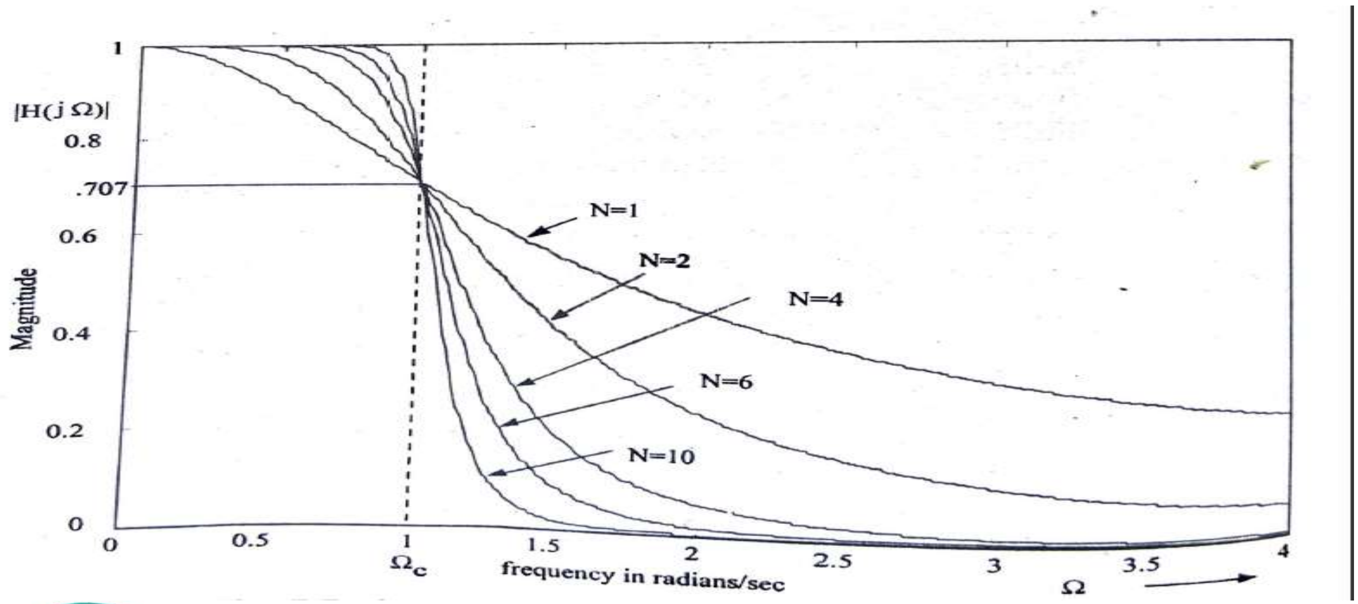
Properties of IIR Filters

Analog Filter Type	**Pass-Band Ripple**	**Stop-Band Ripple**	**Transition Band**
Butterworth	Monotonic (Maximally Flat)	Monotonic	Wide
Chebyshev-I	Equi-ripple	Monotonic	Narrow
Chebyshev-II	Monotonic	Equi-ripple	Narrow

Analog lowpass Butterworth Filter

The magnitude function of the Butterworth lowpass filter is given by

$$|H(j\Omega)| = \frac{1}{\left[1 + (\Omega/\Omega_c)^{2N}\right]^{1/2}} \qquad N = 1, 2, 3, \ldots$$



The following table gives Butterworth polynomials for various values of N for $\Omega_c = 1$ rad/sec.

List of Butterworth Polynomials

N	Denominator of $H(s)$
1	$s + 1$
2	$s^2 + \sqrt{2}s + 1$
3	$(s + 1)(s^2 + s + 1)$
4	$(s^2 + 0.76537s + 1)(s^2 + 1.8477s + 1)$
5	$(s + 1)(s^2 + 0.61803s + 1)(s^2 + 1.61803s + 1)$
6	$(s^2 + 1.931855s + 1)(s^2 + \sqrt{2}s + 1)(s^2 + 0.51764s + 1)$
7	$(s + 1)(s^2 + 1.80194s + 1)(s^2 + 1.247s + 1)(s^2 + 0.445s + 1)$

For fourth order Butterworth filter the transfer function is given by

$$H(s) = \frac{1}{(s^2 + 0.76536s + 1)(s^2 + 1.84776s + 1)}$$

Example 5.1 Given the specification $\alpha_p = 1\,\text{dB}$; $\alpha_s = 30\,\text{dB}$; $\Omega_p = 200\,\text{rad/sec}$; $\Omega_s = 600\,\text{rad/sec}$. Determine the order of the filter.

Solution

From Eq. (5.25)

$$A = \frac{\lambda}{\varepsilon} = \left(\frac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}\right)^{0.5}$$

$$= \left(\frac{10^3 - 1}{10^{0.1} - 1}\right)^{0.5} = 62.115$$

From Eq. (5.26)

$$k = \frac{\Omega_p}{\Omega_s} = \frac{200}{600} = \frac{1}{3}$$

From Eq. (5.27)

$$N \geq \frac{\log A}{\log 1/k}$$

$$\geq \frac{\log 62.115}{\log 3} = 3.758$$

Rounding off N to the next higher integer we get $N = 4$.

Example 5.2 Determine the order and the poles of lowpass Butterworth filter that has a 3 dB attenuation at 500 Hz and an attenuation of 40 dB at 1000 Hz.

Solution

Given data $\alpha_p = 3\,\text{dB}$; $\alpha_s = 40\,\text{dB}$; $\Omega_p = 2 \times \pi \times 500 = 1000\pi$ rad/sec.
$\Omega_s = 2 \times \pi \times 1000 = 2000\pi$ rad/sec.

The order of the filter

$$N \geq \frac{\log \sqrt{\dfrac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\log \dfrac{\Omega_s}{\Omega_p}}$$

$$\geq \frac{\log \sqrt{\dfrac{10^4 - 1}{10^{0.3} - 1}}}{\log \dfrac{2000\pi}{1000\pi}} = 6.6$$

Rounding 'N' to nearest higher value we get $N = 7$.
The poles of Butterworth filter are given by

$$s_k = \Omega_c e^{j\phi_k} = 1000\pi e^{j\phi_k} \quad k = 1, 2, \ldots 7$$

where $\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \quad k = 1, 2, \ldots 7.$