SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
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DEPARTMENT OF BIOMEDICAL ENGINEERING

19BMB302 - BIOMEDICAL SIGNAL PROCESSING

III YEAR/ V SEMESTER

UNIT III INFINITE IMPULSE RESPONSE FILTERS

UNIT II INFINITE IMPULSE RESPONSE FILTERS

Characteristics of practical frequency selective filters.
Characteristics of commonly used analog filters
Butterworth filters, Chebyshev filters.
Design of IIR filters from analog filters (LPF, HPF, BPF, BRF)
Approximation of derivatives
Impulse invariance method
Bilinear transformation
Frequency transformation in the analog domain
Structure of IIR filter - direct form I, direct form II
Cascade, parallel realizations

Example 5.7 Obtain an analog Chebyshev filter transfer function that satisfies the constraints $\frac{1}{\sqrt{2}} \leq |H(j\Omega)| \leq 1; \quad 0 \leq \Omega \leq 2$

$$|H(j\Omega)| < 0.1; \quad \Omega \geq 4$$

Solution

Step 1: From the given data we can find that

$$\frac{1}{\sqrt{1+\varepsilon^2}} = \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{1+\lambda^2}} = 0.1,$$

$\Omega_p = 2$ and $\Omega_s = 4$, from which we can obtain $\varepsilon = 1$ and $\lambda = 9.95$.

We know

$$N \geq \frac{\cosh^{-1}\frac{\lambda}{\varepsilon}}{\cosh^{-1}\frac{\Omega_s}{\Omega_p}} = \frac{\cosh^{-1} 9.95}{\cosh^{-1} 2} = 2.269$$

Step 2: Rounding N to next higher value we get $N = 3$. For N odd, the ... latory curve starts from unity as shown in Fig. 5.12.

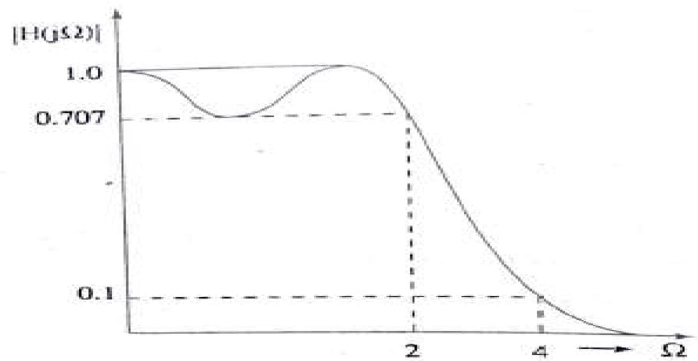


Fig. 5.12 Magnitude response of example 5.7.

Step 3: Finding the values of a and b

$$\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}} = 2.414$$

$$a = \Omega_p \left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right] = 2\left[\frac{(2.414)^{1/3} - (2.414)^{-1/3}}{2}\right]$$
$$= 0.596$$

$$b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right] = 2\left[\frac{(2.414)^{1/3} + (2.414)^{-1/3}}{2}\right]$$
$$= 2.087$$

Step 4: To calculate the poles of Chebyshev filter

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \quad k = 1, 2, 3$$
$$\phi_1 = 120°, \ \phi_2 = 180°, \ \phi_3 = 240°$$

We know $s_k = a\cos\phi_k + jb\sin\phi_k \quad k = 1, 2, 3$ from which we get

$$s_1 = a\cos\phi_1 + jb\sin\phi_1 = 0.596\cos 120° + j2.087\sin 120° = -0.298 + j1.807$$
$$s_2 = a\cos\phi_2 + jb\sin\phi_2 = 0.596\cos 180° + j2.087\sin 180° = -0.596$$
$$s = a\cos\phi_3 + jb\sin\phi_3 = 0.596\cos 240° + j2.087\sin 240° = -0.298 - j1.807$$

Step 5: The denominator polynomial is given by

$$(s+0.596)\{(s+0.298) - j1.807\}\{(s+0.298) + j1.807\}$$
$$= (s+0.596)[(s+0.298)^2 + (1.807)^2]$$
$$= (s+0.596)(s^2 + 0.596s + 3.354)$$

Step 6: The numerator of $H(s)$ can be obtained by substituting $s = 0$ (for N odd) in the denominator.

Therefore the numerator of $H(s) = 2$

The transfer function of Chebyshev filter for the given specifications is given by

$$H(s) = \frac{2}{(s + 0.596)(s^2 + 0.596s + 3.354)}$$

Example 5.8 Determine the order and the poles of a type I lowpass Chebyshev filter that has a 1 dB ripple in the passband and passband frequency $\Omega_p = 1000\pi$, a stopband frequency of 2000π and an attenuation of 40 dB or more.

Solution

Given data $\alpha_p = 1$ dB; $\Omega_p = 1000\pi$ rad/sec; $\alpha_s = 40$ dB
$\Omega_s = 2000\pi$ rad/sec

$$N \geq \frac{\cos h^{-1}\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cos h^{-1}\dfrac{\Omega_s}{\Omega_p}} \geq \frac{\cos h^{-1}\sqrt{\dfrac{10^4-1}{10^{0.1}-1}}}{\cos h^{-1}\dfrac{2000\pi}{1000\pi}} = 4.536$$

i.e., $N = 5$

$$\varepsilon = \sqrt{10^{0.1\alpha_p}-1} = 0.508; \quad \mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}} = 4.17$$

$$a = \Omega_p\left[\frac{\mu^{1/N}-\mu^{-1/N}}{2}\right] = 289.5\pi; \quad b = \Omega_p\left[\frac{\mu^{1/N}+\mu^{-1/N}}{2}\right] = 1041\pi$$

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \quad k = 1, 2, \ldots 5$$

$$\phi_1 = 180°;\ \phi_2 = 144°;\ \phi_3 = 180°;\ \phi_4 = 216°;\ \phi_5 = 252°$$

$$s_k = a\cos\phi_k + jb\sin\phi_k \quad k = 1, 2, \ldots 5$$

$$s_1 = -89.5\pi + j989\pi; \quad s_2 = -234.2\pi + j612\pi; \quad s_3 = -289.5\pi$$

$$s_4 = -234.2\pi - j612\pi; \quad s_5 = -89.5\pi - j989\pi$$

Example 5.9 Design a Chebyshev filter with a maximum passband attenuation of 2.5 dB; at $\Omega_p = 20$ rad/sec and the stopband attenuation of 30 dB at $\Omega_s = 50$ rad/sec.

Solution

Given

$$\Omega_p = 20 \quad \text{rad/sec}; \quad \alpha_p = 2.5\,\text{dB};$$
$$\Omega_s = 50 \quad \text{rad/sec}; \quad \alpha_s = 30\,\text{dB};$$

We know

$$N = \frac{\cosh^{-1} \lambda/\varepsilon}{\cosh^{-1} 1/k}$$
$$\lambda = \sqrt{10^{0.1\alpha_s} - 1} = 31.607$$
$$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1} = 0.882$$
$$k = \frac{\Omega_p}{\Omega_s} = 0.4$$

Now

$$N \geq \frac{\cosh^{-1} \frac{31.607}{0.882}}{\cosh^{-1} \frac{1}{0.4}} = 2.726$$

i.e., $N = 3$

$$\mu = \varepsilon^{-1} + \sqrt{1 + \varepsilon^{-2}} = 2.65$$

$$a = \Omega_p \frac{[\mu^{1/N} - \mu^{-1/N}]}{2} = 6.6$$

$$b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right] = 21.06$$

$$s_k = a\cos\phi_k + jb\sin\phi_k; \quad k = 1, 2, 3$$

$$\phi_k = \frac{\pi}{2} + \left(\frac{2k-1}{2N}\right)\pi; \quad k = 1, 2, 3$$

$$\phi_1 = 120^\circ, \phi_2 = 180^\circ, \phi_3 = 240^\circ$$

$$s_1 = -3.3 + j18.23$$

$$s_2 = -6.6$$

$$s_3 = -3.3 - j18.23$$

Denominator of $H(s) = (s + 6.6)(s^2 + 6.6s + 343.2)$

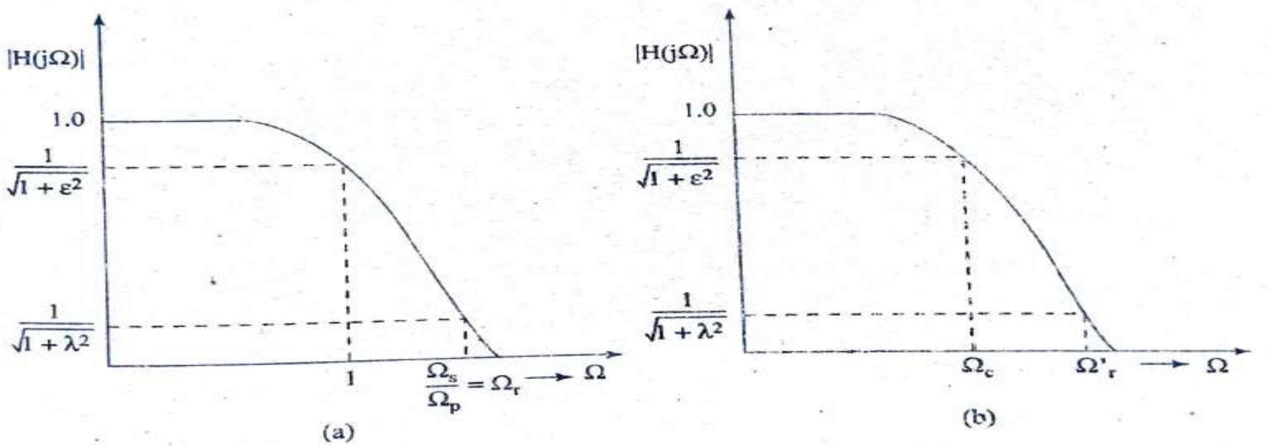
Numerator of $H(s) = (6.6)(343.2) = 2265.27$

Transfer function $H(s) = \dfrac{2265.27}{(s + 6.6)(s^2 + 6.6s + 343.2)}$

5.10.1 *Lowpass to Lowpass Filter*

Given a normalized lowpass filter, it is desirable to have a lowpass filter with a different cutoff frequency Ω_c (or passband frequency Ω_p). This can be accomplished by the transformation given in Eq. (5.62a)

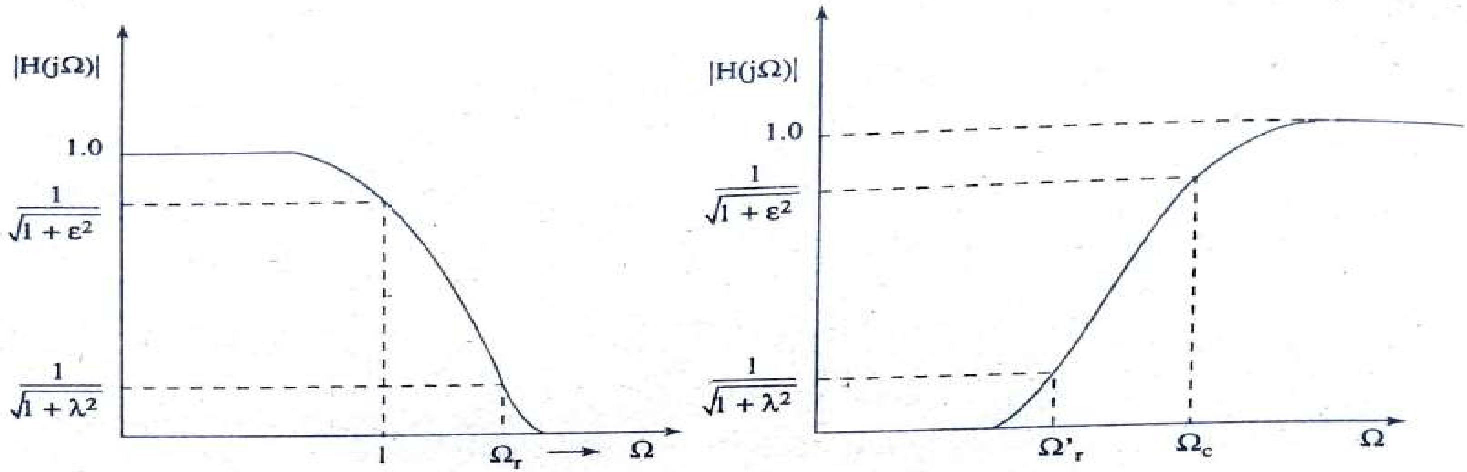
$$s \rightarrow \frac{s}{\Omega_c} \tag{5.62a}$$



5.10.2 *Lowpass to Highpass*

Given a normalized lowpass filter, it is desirable to have a highpass filter with cutoff frequency Ω_c. Then the transformation is

$$s \rightarrow \frac{\Omega_c}{s} \qquad (5.62b)$$



(a) (b)

5.10.3 *Lowpass to Bandpass*

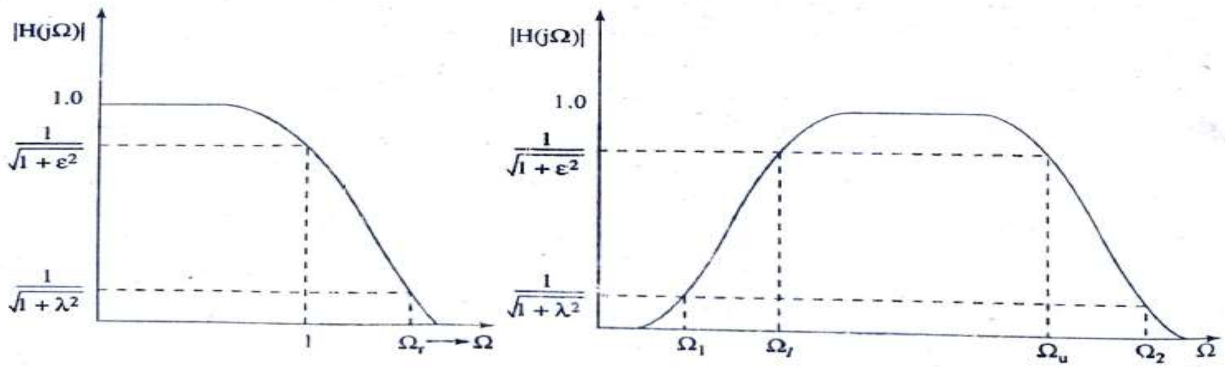
The transformation for converting a normalized lowpass filter to a bandpasss filter with cutoff frequencies Ω_l, Ω_u can be accomplished by

$$s \rightarrow \frac{s^2 + \Omega_l \Omega_u}{s(\Omega_u - \Omega_l)} \quad (5.63a)$$

$$\Omega_r = \min\{|A|, |B|\} \quad (5.63b)$$

$$A = \frac{-\Omega_1^2 + \Omega_l \Omega_u}{\Omega_1(\Omega_u - \Omega_l)} \quad (5.63c)$$

$$B = \frac{\Omega_2^2 - \Omega_l \Omega_u}{\Omega_2(\Omega_u - \Omega_l)} \quad (5.63d)$$



5.10.4 *Lowpass to Bandstop*

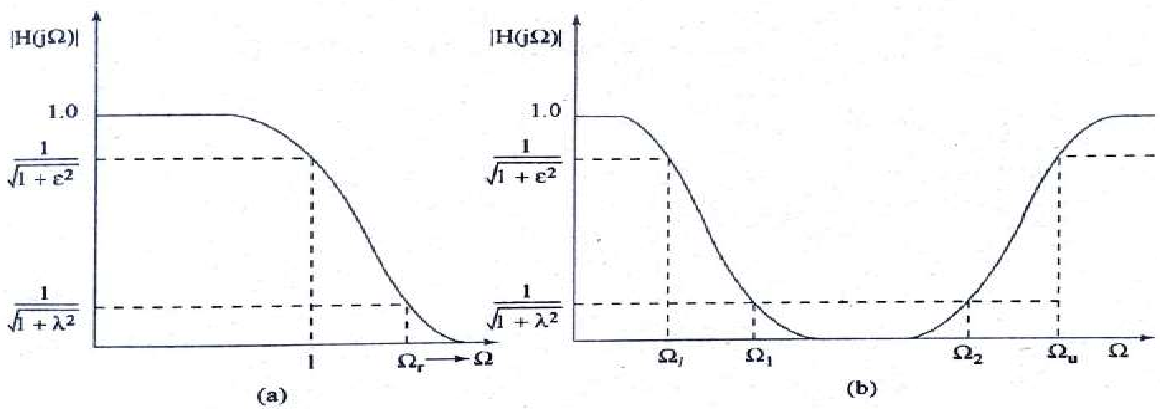
The transformation to convert a normalized lowpass filter to a bandstop filter is

$$s \longrightarrow \frac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_l \Omega_u}$$

$$\Omega_r = \min\{|A|, |B|\} \qquad (5.63e)$$

$$A = \frac{\Omega_1(\Omega_u - \Omega_l)}{-\Omega_1^2 + \Omega_l \Omega_u} \qquad (5.63f)$$

$$B = \frac{\Omega_2(\Omega_u - \Omega_l)}{-\Omega_2^2 + \Omega_l \Omega_u} \qquad (5.63g)$$



Example 5.10 For the given specifications $\alpha_p = 3$ dB; $\alpha_s = 15$ dB; $\Omega_p = 1000$ rad/sec and $\Omega_s = 500$ rad/sec design a highpass filter.

Solution

First we design a normalized lowpass filter and then use suitable transformation to get the transfer function of a highpass filter.

For lowpass filter

$\Omega_c = \Omega_p = 500$ rad/sec

$\Omega_s = 1000$ rad/sec

For highpass filter

$\Omega_c = \Omega_p = 1000$ rad/sec

$\Omega_s = 500$ rad/sec

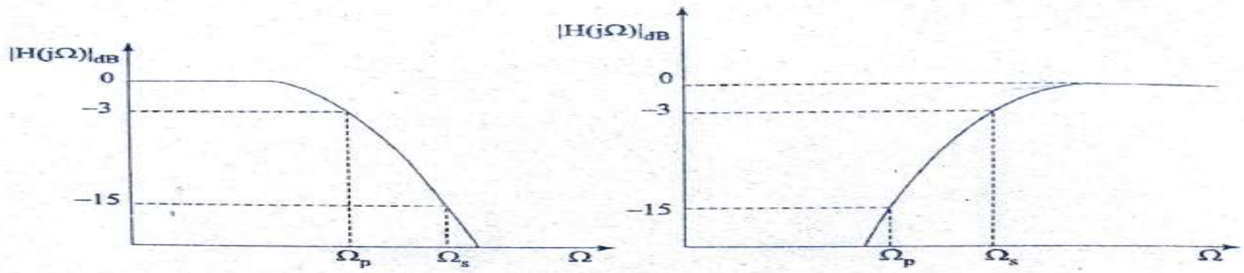


Fig. 5.17 Lowpass to highpass transformation

Lowpass filter specifications

$$\Omega_c = \Omega_p = 500 \text{ rad/sec}; \quad \alpha_p = 3 \text{ dB}$$
$$\Omega_s = 1000 \text{ rad/sec}; \quad \alpha_s = 15 \text{ dB}$$

We have

$$N = \frac{\log \frac{\lambda}{\varepsilon}}{\log 1/k}$$

$$\lambda = \sqrt{10^{0.1\alpha_s} - 1} = 5.533$$

$$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1} = 1$$

$$k = \frac{\Omega_p}{\Omega_s} = 0.5$$

Therefore $N = \frac{\log 5.533}{\log 2} = 2.468$. Approximating to next higher integer we have $N = 3$.

$H(s)$ for $\Omega_c = 1$ rad/sec and $N = 3$ is

$$H(s) = \frac{1}{(s+1)(s^2+s+1)}$$

To get highpass filter having cutoff frequency

$$\Omega_c = \Omega_p = 1000 \text{ rad/sec}$$

Substitute $s \longrightarrow \frac{1000}{s}$

$$H_a(s) = H(s)\Big|_{s \to \frac{1000}{s}}$$

$$= \frac{1}{(s+1)(s^2+s+1)}\Bigg|_{s \to \frac{1000}{s}}$$

$$= \frac{s^3}{(s+1000)[s^2+1000s+(1000)^2]}$$