



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



DEPARTMENT OF MATHEMATICS

23MAT101 - MATRICES AND CALCULUS

UNIT-III DIFFERENTIAL CALCULUS

Evolute:

The locus of the centre of curvature of the given curve is called the evolute of the curve.

Working procedure to find the evolute:

1. Write the parametric form of the given curve.
2. Find centre of curvature (\bar{x}, \bar{y})
3. Eliminate the parameter from \bar{x} & \bar{y}
4. Taking locus of the above equation, we get the required evolute.

Curve	Cartesian Equation	Parametric Equation.
Parabola	① $y^2 = 4ax$ ② $x^2 = 4ay$	① $x = at^2, y = 2at$ ② $x = 2at, y = at^2$
Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$x = a \cos \theta, y = b \sin \theta$
Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$x = a \sec \theta, y = b \tan \theta$
Rectangular hyperbola	$xy = c^2$	$x = ct, y = \frac{c}{t}$
Astroid	$x^{2/3} + y^{2/3} = a^{2/3}$	$x = a \cos^3 \theta,$ $y = a \sin^3 \theta.$



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Problems:

① Find the equation of the evolute of the parabola

$$y^2 = 4ax$$

Soln:-

The parametric equations of the parabola

$$y^2 = 4ax \text{ are } x = at^2, y = 2at$$

$$\frac{dx}{dt} = 2at, \frac{dy}{dt} = 2a$$

$$y_1 = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2a}{2at} = \frac{1}{t}$$

$$y_2 = \frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$$

$$= \frac{d}{dx} \left(\frac{1}{t} \right)$$

$$= \frac{d}{dx} (t^{-1})$$

$$= -\frac{1}{t^2} \cdot \frac{dt}{dx}$$

$$= -\frac{1}{t^2} \cdot \frac{1}{dx/dt}$$

$$= -\frac{1}{t^2} \cdot \frac{1}{2at}$$

$$y_2 = \frac{-1}{2at^3}$$

$$\bar{x} = x - \frac{y_1}{y_2} (1 + y_1^2)$$

$$= at^2 - \frac{1/t}{-1/2at^3} \left(1 + \frac{1}{t^2} \right)$$



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$$= at^2 + \frac{1}{t} \cdot 2at^3 \left(1 + \frac{1}{t^2}\right)$$

$$= at^2 + 2at^2 \left(1 + \frac{1}{t^2}\right)$$

$$= at^2 + 2at^2 + 2at^2 \cdot \frac{1}{t^2}$$

$$= at^2 + 2at^2 + 2a$$

$$\bar{x} = 3at^2 + 2a \Rightarrow t^2 = \frac{\bar{x} - 2a}{3a} \rightarrow \textcircled{1}$$

$$\bar{y} = y + \frac{1}{y} (1 + y^2)$$

$$= 2at + \frac{1}{-1/2at^3} \left(1 + \frac{1}{t^2}\right)$$

$$= 2at - 2at^3 \left(1 + \frac{1}{t^2}\right)$$

$$= 2at - 2at^3 - 2at^3 \cdot \frac{1}{t^2}$$

$$= 2at - 2at^3 - 2at$$

$$y^3 = -2at^3$$

$$t^3 = \frac{-y}{2a} \rightarrow \textcircled{2}$$

Taking cube of $\textcircled{1}$ & squaring $\textcircled{2}$ we get,

$$(t^2)^3 = \left(\frac{\bar{x} - 2a}{3a}\right)^3$$

$$t^6 = \frac{(\bar{x} - 2a)^3}{27a^3} \rightarrow \textcircled{3}$$



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$$(t^3)^2 = \left(\frac{-y}{2a}\right)^2$$

$$t^6 = \frac{y^2}{4a^2} \rightarrow \textcircled{4}$$

From $\textcircled{3}$ & $\textcircled{4}$

$$\frac{(\bar{x} - 2a)^3}{27a^3} = \frac{y^2}{4a^2}$$

$$\frac{(\bar{x} - 2a)^3}{27a} = \frac{y^2}{4}$$

$$4(\bar{x} - 2a)^3 = 27ay^2$$

Locus of (\bar{x}, \bar{y}) is

$$\boxed{4(x - 2a)^3 = 27ay^2}$$

which is the evolute of the parabola

$$y^2 = 4ax$$