



DEPARTMENT OF MATHEMATICS

23MAT101 - MATRICES AND CALCULUS

UNIT-I MATRIX EIGENVALUE PROBLEM

⇒ Cayley Hamilton theorem Period 4

Every square matrix satisfies its own characteristic equation.

⇒ uses of Cayley Hamilton theorem:-

1) positive integral powers of A

2) A^{-1} ⇒ Inverse of A

1) Verify Cayley Hamilton theorem for

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \text{ find } A^4 \text{ and } A^{-1}$$

Soln:-

The characteristic equation

$$\lambda^3 - D_1\lambda^2 + D_2\lambda - D_3 = 0$$

$$D_1 = 2 + 2 + 2 = 6$$

$$D_2 = (4 - 1) + (4 - 1) + (4 - 1)$$

$$= 3 + 3 + 3 = 9$$

$$D_3 = 2(4 - 1) - (-1)(-2 + 1) + 1(1 - 2)$$

$$= 2(3) + 1(-1) + 1(-1)$$

$$= 6 - 1 - 1 = 4$$

$$\therefore \lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$$

By Cayley Hamilton theorem,

$$A^3 - 6A^2 + 9A - 4I = 0$$

$$\Rightarrow A^3 - 6A^2 + 9A - 4I = 0$$

To find A^4, A^5



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Now

$$A^2 = A \times A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^3 = A^2 \times A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

Now

$$A^3 - 6A^2 + 9A - 4I = 0$$

$$\begin{pmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{pmatrix} - 6 \begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix} + 9 \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix} - 4 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 0$$

$$= \begin{pmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{pmatrix} - \begin{pmatrix} 36 & -30 & 30 \\ 30 & 36 & -30 \\ 30 & -30 & 36 \end{pmatrix} + \begin{pmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{pmatrix} - \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} = 0$$

$$= \begin{pmatrix} 22-36+18-4 & -21+30-9 & 21-30+9 \\ -21+30-9 & 22-36+18-4 & -21+30-9 \\ 21-30+9 & -21+30-9 & 22-36+18-4 \end{pmatrix}$$



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$$A^3 - 6A^2 + 9A - 4I = 0$$

$$\Rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = 0$$

To find A^4

$$A^3 - 6A^2 + 9A - 4I = 0 \quad (\because IA = A)$$

$$A^3 = 6A^2 - 9A + 4I$$

$$A^4 = 6A^3 - 9A^2 + 4A$$

$$A^4 = 6 \begin{pmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{pmatrix} - 9 \begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix} + 4 \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 132 & -126 & 126 \\ -126 & 132 & -126 \\ 126 & -126 & 132 \end{pmatrix} - \begin{pmatrix} 54 & -45 & 45 \\ -45 & 54 & -45 \\ 45 & -45 & 54 \end{pmatrix} + \begin{pmatrix} 8 & -4 & 4 \\ -4 & 8 & -4 \\ 4 & -4 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 132 - 54 + 8 & -126 + 45 - 4 & 126 - 45 + 4 \\ -126 + 45 - 4 & 132 - 54 + 8 & -126 + 45 - 4 \\ 126 - 45 + 4 & -126 + 45 - 4 & 132 - 54 + 8 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 86 & -85 & 85 \\ -85 & 86 & -85 \\ 85 & -85 & 86 \end{pmatrix}$$

To find A^{-1}

$$A^3 - 6A^2 + 9A - 4I = 0 \quad (\because A^{-1}A = I)$$

$$A^2 - 6A + 9I - 4A^{-1} = 0 \quad (\because IA^{-1} = A^{-1})$$

$$A^{-1} = \frac{1}{4} [A^2 - 6A + 9I]$$

$$A^{-1} = \frac{1}{4} \left[\begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix} - 6 \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix} + 9 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$



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$$A^{-1} = \frac{1}{4} \left[\begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix} - \begin{pmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{pmatrix} + \begin{pmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix} \right]$$

$$A^{-1} = \frac{1}{4} \begin{pmatrix} 6-12+9 & -5+6+0 & 5-6+0 \\ -5+6+0 & 6-12+9 & -5+6+0 \\ 5-6+0 & -5+6+0 & 6-12+9 \end{pmatrix}$$

$$A^{-1} = \frac{1}{4} \begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 3/4 & 1/4 & -1/4 \\ 1/4 & 3/4 & 1/4 \\ -1/4 & 1/4 & 3/4 \end{pmatrix}$$

2. Verify Cayley Hamilton theorem for $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$, find A^4 and A^{-1} Period: 3

Soln:- The characteristic equation

$$\lambda^3 - D_1 \lambda^2 + D_2 \lambda - D_3 = 0$$

$$D_1 = 1+1+1 = 3$$

$$D_2 = (1-1) + (1-3) + (1-0)$$

$$= 0 - 2 + 1 = -1$$

$$D_3 = 1(1-1) - 0 + 3(-2-1)$$

$$= 3(-3) = -9$$

$$\therefore \lambda^3 - 3\lambda^2 - \lambda + 9 = 0$$

By Cayley Hamilton theorem

$$A^3 - 3A^2 - A + 9I = 0$$

$$\therefore A^3 - 3A^2 - A + 9I = 0$$

To find: A^2, A^3



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Now

$$- A^2 = A \times A$$

$$A^2 = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 4 & -3 & 6 \\ 8 & 2 & 4 \\ 0 & -2 & 5 \end{pmatrix}$$

$$A^3 = A^2 \times A$$

$$= \begin{pmatrix} 4 & -3 & 6 \\ 8 & 2 & 4 \\ 0 & -2 & 5 \end{pmatrix} \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 21 & -9 & 21 \\ 11 & -2 & 11 \\ 1 & -7 & 7 \end{pmatrix}$$

Now:-

$$A^3 - 3A^2 - A + 9I = 0$$

$$\begin{pmatrix} 21 & -9 & 21 \\ 11 & -2 & 11 \\ 1 & -7 & 7 \end{pmatrix} - 3 \begin{pmatrix} 4 & -3 & 6 \\ 8 & 2 & 4 \\ 0 & -2 & 5 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix} + \begin{pmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -9 & 21 \\ 11 & -2 & 11 \\ 1 & -7 & 7 \end{pmatrix} - \begin{pmatrix} 12 & -9 & 18 \\ 24 & 6 & 12 \\ 0 & -6 & 15 \end{pmatrix} + \begin{pmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix}$$



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$$= \begin{pmatrix} 4-12-1+9 & -9+9-0+0 & 21-18-3+0 \\ 11-9-2+0 & 2-6-1+9 & 11-12+1+0 \\ 1-0-1+0 & -7+6+1+0 & 7-15-1+9 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = 0$$

To find A^4, A^{-1}

$$A^3 - 3A^2 - A + 9I = 0$$

$$A^4 - 3A^3 + A^2 - 9A,$$

$$A^4 = 3 \begin{pmatrix} 4 & -9 & 21 \\ 11 & -2 & 11 \\ 1 & -7 & 7 \end{pmatrix} + \begin{pmatrix} 11 & -9 & 6 \\ 8 & 0 & 4 \\ 0 & -2 & 5 \end{pmatrix} - 9 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 12 & -27 & 21 \\ 33 & -6 & 33 \\ 3 & -21 & 21 \end{pmatrix} + \begin{pmatrix} 11 & -9 & 6 \\ 8 & 0 & 4 \\ 0 & -2 & 5 \end{pmatrix} - \begin{pmatrix} 9 & 0 & 9 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 12+11-9 & -27-9-0 & 21+6-9 \\ 33+8-0 & -6+0-9 & 33+4+0 \\ 3+0-9 & -21-2+0 & 21+5-9 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & -36 & 18 \\ 41 & -15 & 37 \\ -6 & -23 & 17 \end{pmatrix}$$

To find A^{-1}

$$A^2 - 3A - I + 9A^{-1} = 0$$

$$A^{-1} = [A^2 + 3A + I]^{-1} / 9$$

$$A^{-1} = \frac{1}{9} \left[\begin{pmatrix} 4 & -9 & 21 \\ 11 & -2 & 11 \\ 1 & -7 & 7 \end{pmatrix} + 3 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$



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$$A^{-1} = \left[\begin{pmatrix} -4 & 3 & -6 \\ -3 & 2 & -4 \\ 0 & 2 & -5 \end{pmatrix} + \begin{pmatrix} 3 & 0 & 3 \\ 6 & 3 & -2 \\ 8 & -3 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

$$A^{-1} = \begin{bmatrix} -4+3+1 & 3+0+0 & -6+3+0 \\ -3+6+0 & -2+3+1 & -4-3+0 \\ 0+3+0 & 2-3+0 & -5+3+1 \end{bmatrix}$$

$$A^{-1} = \begin{pmatrix} 0 & 3 & -3 \\ 3 & 2 & -7 \\ 3 & -1 & -1 \end{pmatrix} \frac{1}{9}$$

$$A^{-1} = \frac{1}{9} \begin{pmatrix} 0 & 3 & -3 \\ 3 & 2 & -7 \\ 3 & -1 & -1 \end{pmatrix}$$

3) Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$,
Find A^3 and A^{-1} .

$$\text{Let } A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

The characteristic equation is

$$\lambda^2 - D_1\lambda + D_2 = 0$$

$$D_1 = 1+3 = 4$$

$$D_2 = |A|$$

$$= \begin{vmatrix} 1 & 4 \\ 2 & 3 \end{vmatrix} = 3-8 = -5$$

$$\therefore \lambda^2 - 4\lambda - 5 = 0$$

By Cayley Hamilton theorem

$$A^2 - 4A - 5I = 0$$

$$A^2 = A \times A$$

$$= \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$



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$$\begin{aligned} A^2 &= \begin{bmatrix} 9 & 16 \\ 8 & 17 \end{bmatrix} - 4 \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 9 & 16 \\ 8 & 17 \end{bmatrix} - \begin{bmatrix} 4 & 16 \\ 8 & 12 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 9-4-5 & 16-16-0 \\ 8-8-0 & 17-12-5 \end{bmatrix} \end{aligned}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

To find A^3, A^{-1}

$$A^2 - 4A - 5I = 0$$

$$A^3 = 4A^2 - 5A$$

$$A^3 = 4 \begin{bmatrix} 9 & 16 \\ 8 & 17 \end{bmatrix} - 5 \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 36 & 64 \\ 32 & 68 \end{bmatrix} - \begin{bmatrix} 5 & 20 \\ 10 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 36-5 & 64-20 \\ 32-10 & 68-15 \end{bmatrix}$$

$$= \begin{bmatrix} 31 & 44 \\ 22 & 53 \end{bmatrix}$$

$$A^2 - 4A - 5I = 0$$

$$A - 4I - 5A^{-1} = 0$$

$$A^{-1} = \frac{1}{5} [A - 4I]$$



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$$A^{-1} = \frac{1}{5} \left[\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} - 4 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right]$$

$$A^{-1} = \frac{1}{5} \left[\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} - \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \right]$$

$$A^{-1} = \frac{1}{5} \left[\begin{pmatrix} 1-4 & 4-0 \\ 2-0 & 3-4 \end{pmatrix} \right]$$

$$A^{-1} = \frac{1}{5} \begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}$$

4) Use Cayley Hamilton theorem, to find the value of $A^2 - 5A^T + 7A^6 - 3A^5 + A^4 - 2A^3 + 2A^2 - 3A + I = 0$
 where $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$

The characteristic equation

$$\lambda^3 - D_1 \lambda^2 + D_2 \lambda - D_3 = 0$$

$D_1 =$ sum of the main diagonal elements.

$$D_1 = 2 + 1 + 2$$

$$D_1 = 5$$

$D_2 =$ sum of the minors of main diagonal elements.

$$= (2-0) + (4-1) + (2-0)$$

$$= 2 + 3 + 2 = 7$$

$$D_3 = |A|$$

$$= 2(2-0) - 1(0-1) + 1(0-1)$$

$$= 2(2) + 1(-1)$$

$$= 4 - 1 = 3$$



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$\therefore \lambda^5 - 5\lambda^2 + 7\lambda - 3 = 0$
 using Cayley Hamilton theorem -
 $A^5 - 5A^2 + 7A - 3I = 0$
 $A^5 - 5A^3 + 7A^2 - 3A^2 + A^4 - 5A^3 + 8A^2 - 2A + I = 0$

$$\begin{array}{r}
 A^5 + A \\
 \hline
 A^5 - 5A^3 + 7A^2 - 3A^2 + A^4 - 5A^3 + 8A^2 - 2A + I \\
 - 17A - 3I \\
 \hline
 A^4 - 5A^3 + 8A^2 - 2A \\
 A^4 - 5A^3 + 7A^2 - 3A \\
 \hline
 A^2 + A + I
 \end{array}$$

$A^5 - 5A^3 + 7A^2 - 3A^2 + A^4 - 5A^3 + 8A^2 - 2A + I = (A^5 - 5A^3 + 7A - 3I) + (A^2 + A + I)$
 $= 0 + (A^2 + A + I)$
 $= A^2 + A + I$

$A^2 = A \times A$
 $= \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$
 $= \begin{pmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 0 \end{pmatrix}$

$A^2 + A + I = \begin{pmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
 $= \begin{pmatrix} 3 & 5 & 5 \\ 0 & 2 & 0 \\ 2 & 2 & 1 \end{pmatrix}$



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5) Use Cayley Hamilton theorem

$A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$... To express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as linear polynomial in A . And find the value

The characteristic equation is

$$\lambda^2 - D_1\lambda + D_2 = 0$$

$D_1 = 1+3 = 4$

$D_2 = 3-8 = -5$

$\therefore \lambda^2 - 4\lambda - 5 = 0 \Rightarrow A^2 - 4A - 5I = 0$

$\rightarrow A^2 - 4A - 5I =$

using Cayley Hamilton theorem,

$$A^2 - 4A - 5I$$

$$A^3 - 4A^2 - 5A$$

$$A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$$

$$A^5 - 4A^4 - 5A^3$$

$-2A^3 + 11A^2 - A$

$-3A^3 + 6A^2 + 10A$

(+) (+) (-)

$-3A^2 - 11A - 10I$

$3A^2 - 11A - 10I$

(+) (+) (-)

$A + 5I$

$A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I = (A^3 - 2A - 10I)$

$(A^2 - 4A - 5I) + (A + 5I)$

$= 0 + (A + 5I)$

$= (A + 5I)$

$= \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 2 & 8 \end{bmatrix}$