UNIT III
DIFFERENTIAL CALCULUS

UNIT - III

DIFFERENTIAL CALCULUS

I. Basic differentiation formulas :

S.No	y	dy/dx
1	K (constant)	0
2	x^n	$n x^{n-1}$
3	e^{ax}	$a e^{ax}$
4	$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
5	$\log x$	$\frac{1}{x}$
6	a^x	$a^x \log a$
7	$\sin x$	$\cos x$
8	$\cos x$	$-\sin x$
9	$\tan x$	$\sec^2 x$
10	$\cot x$	$-\operatorname{cosec}^2 x$
11	$\sec x$	$\sec x \tan x$
12	$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
13	$\sinh x$	$\cosh x$
14	$\cosh x$	$\sinh x$

II. **Product rule :**

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx} = uv' + vu'$$

Where u & v are functions of x.

III. **Quotient rule :**

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{vu' - uv'}{v^2}$$

IV. If $x = x(t)$, $y = y(t)$ where 't' is a Parameter then,

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}\left(\frac{dy}{dx}\right)\frac{dt}{dt}$$

$$= \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}$$

$$= \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{1}{dx/dt}$$

Curvature of a curve:

The rate of bending of a curve in any interval is called the curvature of the curve in that interval. It is denoted by K.

Note: The curvature of a straight line is zero.

Radius of curvature:

It is defined as the reciprocal of the curvature of the curve and is denoted by ρ.

$$\boxed{\rho = \frac{1}{K}}$$

Note:

The radius of curvature at every point of the circle is equal to the radius of the circle. i.e., $\rho = r$

Hence the curvature is, $K = \frac{1}{\rho} = \frac{1}{r}$

Formula for radius of curvature:

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2} \quad (\text{or}) \quad \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$$

where $y_1 = \frac{dy}{dx}$, $y_2 = \frac{d^2y}{dx^2}$

Note: When $\frac{dy}{dx}$ becomes ∞, $\rho = \frac{\left[1+\left(\frac{dx}{dy}\right)^2\right]^{3/2}}{d^2x/dy^2}$ is the alternative formula for radius of curvature.