



## DEPARTMENT OF MATHEMATICS

### UNIT III

### DIFFERENTIAL CALCULUS

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#### DIFFERENTIAL CALCULUS

#### I. Basic differentiation formulas :

S.No	y	dy/dx
①	K (constant)	0
②	$x^n$	$n x^{n-1}$
③	$e^{ax}$	$a e^{ax}$
④	$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
⑤	$\log x$	$\frac{1}{x}$
⑥	$a^x$	$a^x \log a$
⑦	$\sin x$	$\cos x$
⑧	$\cos x$	$-\sin x$
⑨	$\tan x$	$\sec^2 x$
⑩	$\cot x$	$-\operatorname{cosec}^2 x$
⑪	$\sec x$	$\sec x \tan x$
⑫	$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
⑬	$\sin h x$	$\cosh x$
⑭	$\cos h x$	$\sinh x$



# SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

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## DEPARTMENT OF MATHEMATICS

II. Product rule :

$$\frac{d}{dx} (uv) = u \frac{dv}{dx} + v \frac{du}{dx} = uv' + vu'$$

where  $u$  &  $v$  are functions of  $x$ .

III. Quotient rule :

$$\frac{d}{dx} \left( \frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} = \frac{vu' - uv'}{v^2}$$

IV. If  $x = x(t)$ ,  $y = y(t)$  where ' $t$ ' is a parameter then,

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left( \frac{dy}{dx} \right)$$

$$= \frac{d}{dx} \left( \frac{dy}{dx} \right) \frac{dt}{dt}$$

$$= \frac{d}{dt} \left( \frac{dy}{dx} \right) \frac{dt}{dx}$$

$$= \frac{d}{dt} \left( \frac{dy}{dx} \right) \cdot \frac{1}{dx/dt}$$

## DEPARTMENT OF MATHEMATICS

### Curvature of a curve :

The rate of bending of a curve in any interval is called the curvature of the curve in that interval. It is denoted by  $\kappa$ .

Note : The curvature of a straight line is zero.

### Radius of curvature :

It is defined as the reciprocal of the curvature of the curve and is denoted by  $\rho$ .

$$\rho = \frac{1}{\kappa}$$

Note :

The radius of curvature at every point of the circle is equal to the radius of the circle. i.e,  $\rho = r$

Hence the curvature is,  $\kappa = \frac{1}{\rho} = \frac{1}{r}$

### Formula for radius of curvature :

$$\rho = \frac{(1 + y_1^2)^{3/2}}{y_2} \quad (\text{or}) \quad \frac{[1 + (\frac{dy}{dx})^2]^{3/2}}{d^2y/dx^2}$$

where  $y_1 = \frac{dy}{dx}$ ,  $y_2 = \frac{d^2y}{dx^2}$

Note : When  $\frac{dy}{dx}$  becomes  $\infty$ ,  $\rho = \frac{[1 + (\frac{dx}{dy})^2]^{3/2}}{d^2x/dy^2}$

is the alternative formula for radius of curvature.