VARIGNON's THEOREM

Moment of a force about any point is equal to the sum of the moments of the components of that force about the same point. To prove this theorem, consider the force **R** acting in the plane of the body shown in Figure.1. The forces **P** and **Q** represent any two nonrectangular components of **R.** The moment of **R** about point O is

$\mathbf{M_0} = \mathbf{r} \times \mathbf{R}$

Because **R** = **P**+**Q**, we may write

$\mathbf{r} \times \mathbf{R} = \mathbf{r} \times (\mathbf{P}+\mathbf{Q})$

Using the distributive law for cross products, we have

$\mathbf{M_0} = \mathbf{r} \times \mathbf{R} = \mathbf{r} \times \mathbf{P} + \mathbf{r} \times \mathbf{Q}$

This says that the moment of R about O equals the sum of the moments about O of its components P and Q.

This proves the theorem. Varignon's theorem need not be restricted to the case of two components, but it applies equally well to three or more where we take the clockwise moment sense to be positive.

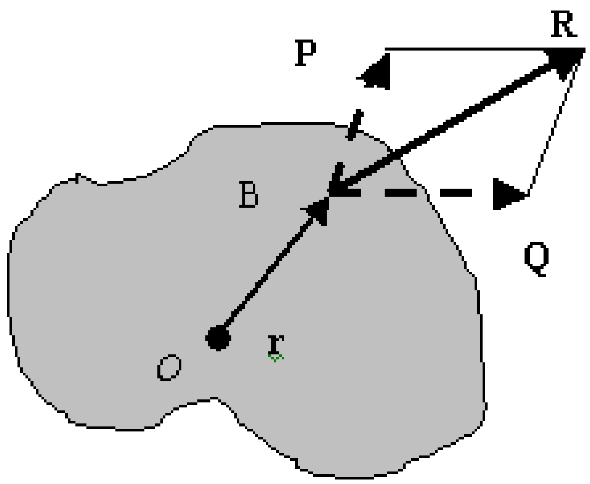


Fig. 11 Illustrating Varignon's theorem

Theorem of Varignon's

The moment of the resultant of two concurrent forces with respect to a centre in their plane is equal to the algebraic sum of the moments of the components with respect to some centre.

Introduction

In our day-to-day work, we see that whenever we apply a force on a body, it exerts a reaction, *e.g.,* when a ceiling fan is hung from a girder, it is subjected to the following two forces:

1. Weight of the fan, acting downwards, and
2. Reaction on the girder, acting upwards.

A little consideration will show, that as the fan is in equilibrium therefore, the above two forces must be equal and opposite. Similarly, if we consider the equilibrium of a girder supported on the walls, we see that the total weight of the fan and girder is acting through the supports of the girder on the walls. It is thus obvious, that walls must exert equal and upward reactions at the supports to maintain the equilibrium. The upward reactions, offered by the walls, are known as support reactions. As a matter of fact, the support reaction depends upon the type of loading and the support.

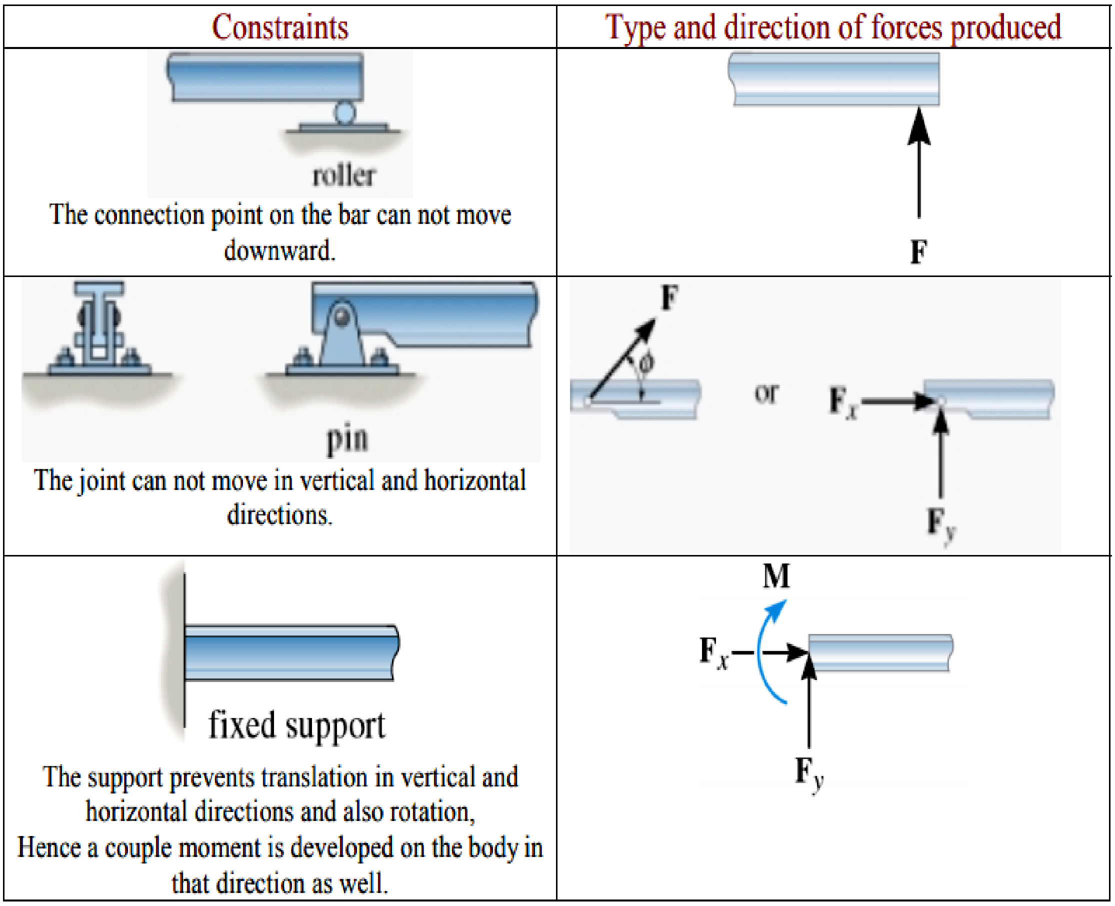


Fig. 12 Supports and Reactions

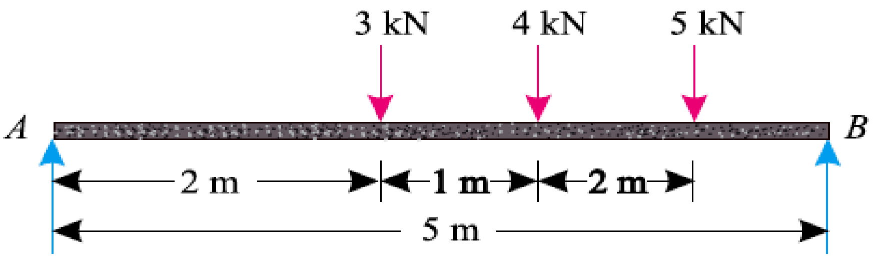
TYPES OF END SUPPORTS OF BEAMS

Though there are many types of supports, for beams and frames, yet the following three types of supports are important from the subject point of view:

1. Simply supported beams,
2. Roller supported beams, and
3. Hinged beams

Worked out examples

A simply supported beam AB of span 5 m is loaded as shown in Figure. Find the reactions at A and B.



Solution:

Given: Span (l) = 5 m

Let R_A = Reaction at A, and

R_B = Reaction at B.

The example may be solved either analytically or graphically. But we shall solve analytically only. We know that anticlockwise moment due to R_B about A

$= R_B \times l = R_B \times 5$

$= 5\ R_B$ kN-m ...(i)

And sum of the clockwise moments about A,

$= (3 \times 2) + (4 \times 3) + (5 \times 4)$

$= 38$ kN-m ...(ii)

Now equating anticlockwise and clockwise moments given in (i) and (ii),

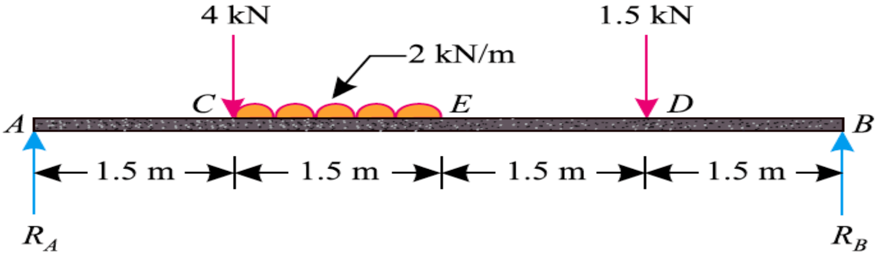
$5\ R_B = 38$

$$R_B = \frac{38}{5} = 7.6 \text{ kN}$$

$$R_A = (3 + 4 + 5) - 7.6 = 4.4 \text{ kN}$$

Worked out examples

A simply supported beam, AB of span 6 m is loaded as shown in Figure. Determine the reactions RA and RB of the beam.



Solution:

Given:

Span (l) = 6m

Let R_A = Reaction at A, and

R_B = Reaction at B.

The example may be solved either analytically or graphically. But we shall solve it analytically only.

We know that anticlockwise moment due to the reaction R_B about A.

$= R_B \times l = R_B \times 6 = 6\ R_B$ KN.m ...(i)

And sum of the clockwise moments about A

$= (4 \times 1.5) + (2 \times 1.5)\ 2.25 + (1.5 \times 4.5)$

$= 19.5$ KN.m ...(ii)

Equating anticlockwise and clockwise moments given in (i) and (ii),

$6\ R_B = 19.5$

$R_B = 19.5 / 6$

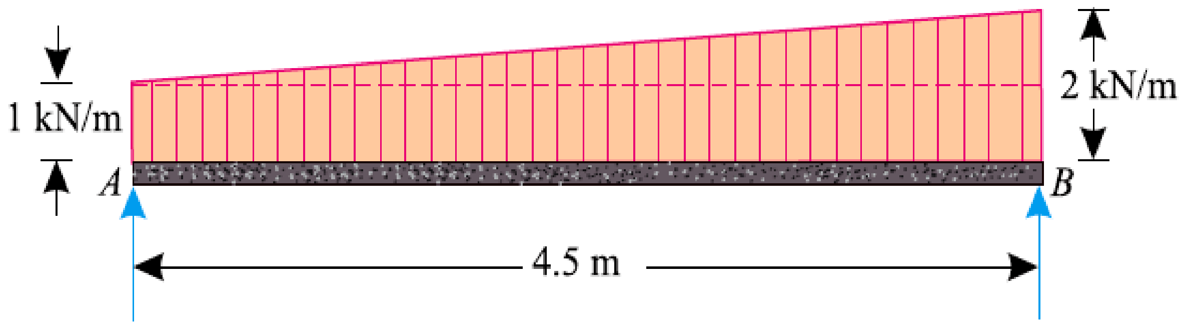
$\boldsymbol{R_B = 3.25}$ **KN**

And $R_A = 4 + (2 \times 1.5) + 1.5 - 3.25$

$\boldsymbol{R_A = 5.25}$ **KN**

Worked out examples

A simply supported beam AB of span 4.5 m is loaded as shown in Figure. Find the support reactions at A and B.



Solution:

Given: Span (l) = 4.5 m

Let R_A = Reaction at A, and

R_B = Reaction at B.

The uniformly distributed load of 2 kN/m for a length of 1.5 m (*i.e.*, between C and E) is assumed as an equivalent point load of $2 \times 1.5 = 3$ kN and acting at the centre of gravity of the load *i.e.*, at a distance of $1.5 + 0.75 = 2.25$ m from A.

The uniformly distributed load of 1 kN/m over the entire span is assumed as an equivalent point load of $1 \times 4.5 = 4.5$ kN and acting at the centre of gravity of the load *i.e.* at a distance of 2.25 m from A. Similarly, the triangular load in assumed as an equivalent point load of $4.5 \times \frac{0+1}{2} = 2.25$ kN and acting at the centre of gravity of the load *i.e.*, distance of $4.5 \times \frac{2}{3} = 3$ m from A.

We know that anticlockwise moment due to R_B about A

$= R_B \times l = R_B \times 4.5 = 4.5\ R_B$ kN-m ...(*i*)

And sum of clockwise moments due to uniformly varying load about A

$= (1 \times 4.5 \times 2.25) + (2.25 \times 3)$

$= 16.875$ kN-m ...(*ii*)

Now equating anticlockwise and clockwise moments given in (*i*) and (*ii*),

$$4.5\ R_B = 16.875$$

$$R_B = \frac{16.875}{4.5} = 3.75 \text{ kN}$$

$$R_A = [1 \times 4.5] + \left[4.5 \times \frac{0+1}{2}\right] - 3.75 = 3.0 \text{ kN}$$

Equivalent Force Couple System

Every set of forces and moments has an **equivalent force couple system**. This is a single force and pure moment (couple) acting at a single point that is **statically equivalent** to the original set of forces and moments.