Properties of Eigen value:

⇒ 1) Sum of the Eigen values of a matrix is equal to the sum of the principle diagonal elements

Sum of the Eigen value = sum of the diagonal element

⇒ 2) the product of the Eigen value of a matrix is equal to its determinant.

Product of Eigen values = Determinant

⇒ 3) Eigen value of a matrix A and A^T are same.

Eigen value of $A = A^T$

⇒ 4) The eigen value of the triangular matrix are the main diagonal elements

Eigen value of Δ^r matrix = diagonal elements.

⇒ 5) If $\lambda_1, \lambda_2, \lambda_3, \ldots \lambda_n$ are the eigen value of a matrix A, then

* $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_2}, \ldots \frac{1}{\lambda_n}$ are the eigen values of A^{-1}

* $K\lambda_1, K\lambda_2, \ldots, K\lambda_n$ are the eigen values of KA

* $\lambda_1^m, \lambda_2^m, \lambda_3^m, \ldots \lambda_n^m$ are the eigen values of A^m

* $\lambda_1 - K, \lambda_2 - K, \ldots \lambda_n - K$ are the eigen values of $A - KI$

* $\frac{|A|}{\lambda}$ is the eigen value of Adj (A)

1. Find the sum and product of eigen values of the matrix $A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$

WKT the sum of the eigen values of a matrix is equal to the sum of the principal diagonal elements

$\Rightarrow$ Sum of the eigen value $= -3$

We know that the product of eigen values of a matrix is equal to its determinant

$\Rightarrow$ product of eigen values $= (-1)(0) - 1(-2) + 1(1+1)$

$= 2 + 2$

$= 4.$

2) Two Eigen values of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ are 3 and 0. Find the third Eigen value.

wkt Sum of the Eigen value = Sum of the diagonal elements

$$\lambda_1 + \lambda_2 + \lambda_3 = 8 + 7 + 3$$

$$3 + 0 + \lambda_3 = 18$$

$$\lambda_3 = 18 - 3 = 15$$

3) Product of 2 Eigen values of a matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the third Eigen value.

wkt product of the Eigen value = determinant

$$\lambda_1 \lambda_2 \lambda_3 = \begin{vmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{vmatrix}$$

$$16\lambda_3 = 6(9-1) - (-2)(-6+2) + 2(2-6)$$

$$= 6(8) + 2(-4) + 2(-4)$$

$$= 48 - 8 - 8 = 32$$

$\lambda_3 = 32/16$

$\boxed{\lambda_3 = 2}$

4. Find the eigen value of Adj(A) if $A=\begin{bmatrix}3 & 2 & 1\\ 0 & 4 & 2\\ 0 & 0 & 1\end{bmatrix}$

WKT Eigen value of triangular matrix = Diagonal elements

$S_1 = 8$

$S_2 = 4+3+12 = 19$

$S_3 = 3(4)-2(0)+1(0) = 12$

$\therefore$ Eigen values are 3, 4 & 1 of A

WKT $\frac{|A|}{\lambda}$ is the eigen value of adj(A) are

$\lambda^2 - 8\lambda^2 + 19\lambda - 12 = 0$

$|A| = 3(4-0)+2(0)+1(0)$

$= 12$

$\Rightarrow \lambda_1 = \frac{12}{3} = 4$

$\lambda_2 = \frac{12}{4} = 3$

$\lambda_3 = \frac{12}{1} = 12$

5) Find the characteristic equation of the matrix $\begin{bmatrix}1 & 2\\ 0 & 2\end{bmatrix}$ and also get its eigen value.

Characteristic equation is given by

$\lambda^2 - S_1\lambda + S_2 = 0.$

S_1 = Sum of the diagonal elements

$= 1+2 = 3$

$S_2 = |A|$

$= 2-0 = 2.$

∴ The characteristic equation is given by

$\lambda^2 - 3\lambda^2 + 2 = 0.$

$\lambda^2 - 2\lambda - \lambda + 2 = 0$

$\Rightarrow (\lambda-2)(\lambda-1) = 0$

$\lambda = 2, 1$

∴ 1, 2 are the eigen values.

Eigen Vectors:-

Let A be a square matrix of order 3 and λ be a scalar. The column matrix $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ satisfying the equation $(A - \lambda I)X = 0$ is called eigen vector or characteristic vectors.