



# SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai  
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &  
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)  
COIMBATORE-641 035, TAMIL NADU



## DEPARTMENT OF MATHEMATICS

Eigen value problems arising from population models : Leslie model

What is the Leslie matrix ?

- Method for representing dynamics of age or size structured populations.
- Combines population processes (births and deaths) into a single model
- Generally applied to populations with annual breeding cycle.
- By convention, we use only female part of population.

Problems :

- ① The Leslie model describes age - specified population growth, as follows. Let the oldest age attained by the females in some animal population be 6 years. Divide the population into three age classes of 2 years each. Let the Leslie matrix be

$$L = \begin{bmatrix} 0 & 2.3 & 0.4 \\ 0.6 & 0 & 0 \\ 0 & 0.3 & 0 \end{bmatrix} = l_{jk}$$



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## DEPARTMENT OF MATHEMATICS

(i) What is the number of females in each class after 2, 4, 6 years if each class initially consists of 500 females?

(ii) For what initial distribution will the number of females in each class change by the same proportion? What is this rate of change?

Soln:

$$(i) \text{ Given : } L = \begin{pmatrix} 0 & 2.3 & 0.4 \\ 0.6 & 0 & 0 \\ 0 & 0.3 & 0 \end{pmatrix}$$

$$\text{Let } X_0 = \begin{pmatrix} 500 \\ 500 \\ 500 \end{pmatrix}$$

After 2 years, the number of females in each class is given by,

$$X_2 = L X_0 = \begin{pmatrix} 0 & 2.3 & 0.4 \\ 0.6 & 0 & 0 \\ 0 & 0.3 & 0 \end{pmatrix} \begin{pmatrix} 500 \\ 500 \\ 500 \end{pmatrix} = \begin{pmatrix} 1350 \\ 300 \\ 150 \end{pmatrix}$$

After 4 years, the number of females in each class is given by,

$$X_4 = L X_2 = \begin{pmatrix} 0 & 2.3 & 0.4 \\ 0.6 & 0 & 0 \\ 0 & 0.3 & 0 \end{pmatrix} \begin{pmatrix} 1350 \\ 300 \\ 150 \end{pmatrix} = \begin{pmatrix} 750 \\ 810 \\ 90 \end{pmatrix}$$



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## DEPARTMENT OF MATHEMATICS

After 6 years, the number of females in each class is given by,

$$X_6 = LX_4 = \begin{pmatrix} 0 & 2.3 & 0.4 \\ 0.6 & 0 & 0 \\ 0 & 0.3 & 0 \end{pmatrix} \begin{pmatrix} 750 \\ 810 \\ 90 \end{pmatrix} = \begin{pmatrix} 1899 \\ 450 \\ 243 \end{pmatrix}$$

$$(ii) \quad L = \begin{pmatrix} 0 & 2.3 & 0.4 \\ 0.6 & 0 & 0 \\ 0 & 0.3 & 0 \end{pmatrix}$$

The characteristic equation is,

$$\lambda^3 - c_1 \lambda^2 + c_2 \lambda - c_3 = 0 \rightarrow (1)$$

$$c_1 = 0 + 0 + 0 = 0 \Rightarrow \boxed{c_1 = 0}$$

$$c_2 = \begin{vmatrix} 0 & 0 \\ 0.3 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0.4 \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 2.3 \\ 0.6 & 0 \end{vmatrix}$$
$$= 0 + 0 + (-1.38)$$

$$\boxed{c_2 = -1.38}$$

$$c_3 = \begin{vmatrix} 0 & 2.3 & 0.4 \\ 0.6 & 0 & 0 \\ 0 & 0.3 & 0 \end{vmatrix}$$

$$= 0(0) - 2.3(0) + 0.4(0.18)$$

$$\boxed{c_3 = 0.072}$$

Subs  $c_1, c_2, c_3$  in (1),



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## DEPARTMENT OF MATHEMATICS

$$\lambda^3 - 0\lambda^2 - 1.38\lambda - 0.072 = 0$$

$$\lambda^3 - 1.38\lambda - 0.072 = 0$$

Solving, we get,

$$\lambda = 1.2, -1.14, -0.05$$

∴ Let  $\lambda = 1.2$  (Take only positive root

To find eigen vector: for growth)

$$(A - \lambda I) X = 0$$

$$(L - \lambda I) X = 0 \quad \text{Here } A = L$$

$$\begin{pmatrix} 0 - \lambda & 2.3 & 0.4 \\ 0.6 & 0 - \lambda & 0 \\ 0 & 0.3 & 0 - \lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow (2)$$

$$\lambda = 1.2$$

$$(2) \Rightarrow \begin{pmatrix} -1.2 & 2.3 & 0.4 \\ 0.6 & -1.2 & 0 \\ 0 & 0.3 & -1.2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Taking first and second row,

$$\begin{array}{cccc} -1.2 & 2.3 & 0.4 & -1.2 \\ 0.6 & -1.2 & 0 & 0.6 \\ \hline & x_3 & x_1 & x_2 \end{array}$$

$$\frac{x_1}{\begin{vmatrix} 2.3 & 0.4 \\ -1.2 & 0 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 0.4 & -1.2 \\ 0 & 0.6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -1.2 & 2.3 \\ 0.6 & -1.2 \end{vmatrix}}$$



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$$\frac{x_1}{0.48} = \frac{x_2}{0.24} = \frac{x_3}{0.06}$$

$$\frac{x_1}{8} = \frac{x_2}{4} = \frac{x_3}{1}$$

$$x_1 = \begin{pmatrix} 8 \\ 4 \\ 1 \end{pmatrix}$$

$$\therefore 8x + 4x + 1x = 1500$$

$$13x = 1500$$

$$x = \frac{1500}{13} = 115.4$$

$$\boxed{x = 115.4}$$

In class I

$$8x = 8(115.4) = 923$$

In class II

$$4x = 4(115.4) = 462$$

In class III

$$1x = 1(115.4) \approx 115$$

The growth rate will be  $\boxed{\lambda = 1.2}$  per  
3 years.