SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Reaccredited by NBA (B.E - CSE, EEE, ECE, Mech&B.Tech.IT)
COIMBATORE-641 035, TAMIL NADU

CHARACTERISTIC EQUATION:

Let A be a given matrix

Let λ be a scalar

The equation $|A - \lambda I| = 0$ is called

Characteristic equation of the matrix A.

1. Find the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$

Solution :-

The characteristic equation is given by

$$\lambda^2 - S_1\lambda + S_2 = 0.$$

$S_1 = $ Sum of the diagonal elements

$= 1 + 4 = 5$

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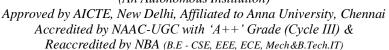*Reaccredited by NBA (B.E - CSE, EEE, ECE, Mech&B.Tech.IT)*
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$S_2 = |A|$

$$= \begin{vmatrix} 1 & 2 \\ -3 & 4 \end{vmatrix} = 4-(-6) = 4+6 = 10$$

$S_2 = 10$

∴ The Characteristic equation is   HW.

$\lambda^2 - 5\lambda + 10 = 0.$

$$A = \begin{bmatrix} 8 & 2 \\ 4 & 7 \end{bmatrix}$$

$S_1 \,\, \textcircled{15}$ $S_2 = 56 - 8$

$S_2 = 48$

2) $\begin{bmatrix} 1 & 1 & 3 \\ 2 & 5 & 6 \\ 4 & 5 & 2 \end{bmatrix}$ Find the characteristic equation

The Characteristic equation is given by

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0.$

$S_1 =$ Sum of the diagonal elements

$= 5+2+1 = 8$

$S_2 =$ Sum of the minors of the diagonal elements

$$= \begin{vmatrix} 5 & 6 \\ 5 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 4 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 2 & 5 \end{vmatrix}$$

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$$= (10 - 30) + (2 - 12) + (5 - 2)$$

$$= -20 - 10 + 3$$

$$S_2 = -27$$

$$S_3 = |A|$$

$$= -20 - 1(4 - 24) + 3(10 - 20)$$

$$= -20 + 20 + 3(-10)$$

$$S_3 = -30$$

The characteristic equation is

$$\lambda^3 - 8\lambda^2 - 27\lambda + 30 = 0.$$

3) $\begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

The characteristic equation is given by

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0.$$

$S_1 = $ Sum of the diagonal elements

$$= 3 + 3 + 3 = 9$$

$S_2 = $ Sum of the minors of the diagonal

elements $= \begin{vmatrix} 3 & -1 \\ -1 & 3 \end{vmatrix} + \begin{vmatrix} 3 & +1 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix}$

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$$= (9-1) + (3+1) + (-1+3)$$

$$= 8 + 8 + 8 = 24$$

$$S_3 = |A|$$

$$= 3(9-1) - 1(3+1) + 1(-1-3)$$

$$= 24 - 4 - 4$$

$$S_3 = 16$$

∴ The characteristic equation is

$$\lambda^3 - 9\lambda^2 + 24\lambda - 16 = 0$$

4)
$$\begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$$

The characteristic equation is given by

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0.$$

S_1 = Sum of the diagonal elements

$$= 7 + 6 + 5$$

$$= 18$$

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S_2 = Sum of the minors of the diagonal elements

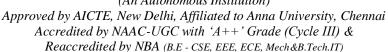$$= \begin{vmatrix} 6 & -2 \\ -2 & 5 \end{vmatrix} + \begin{vmatrix} 7 & 0 \\ 0 & 5 \end{vmatrix} + \begin{vmatrix} 7 & -2 \\ -2 & 6 \end{vmatrix}$$

$$= (30-4) + (35-0) + (42-4)$$

$$= 26 + 35 + 38$$

$$= 99$$

$S_3 = |A|$

$$= 7(30-4) - (-2)(-10-0) + 0(4-0)$$

$$= 7(26) + 2(-10)$$

$$= 182 - 20 = 162$$

∴ The characteristic equation is

$$\lambda^3 - 18\lambda^2 + 99\lambda - 162 = 0.$$

5) Find the characteristic polynomial of [polynomial don't write = 0.

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

The characteristic polynomial of.
$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3$$

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S_1 = Sum of the diagonal elements

$\quad$ = 6 + 3 + 3 $\quad$ = 12

S_2 = Sum of the minors of the diagonal elements

$$= \begin{vmatrix} 3 & -1 \\ -1 & 3 \end{vmatrix} + \begin{vmatrix} 6 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 6 & -2 \\ -2 & 3 \end{vmatrix}$$

$\quad$ = 8 + 14 + 14 $\quad$ = 36

S_3 = |A|

$\quad$ = 6(8) + 2(-6+2) + 2(2-6)

$\quad$ = 48 - 8 - 8

S_3 = 32

∴ The <u>Characteristic polynomial</u> is $\quad$ ↳ defn The determinant

$$\boxed{\lambda^3 - 12\lambda^2 + 36\lambda - 32}$$ = |A-λI| when expanded
$\qquad\qquad\qquad\qquad$ will give a polynomial, which
$\qquad\qquad\qquad\qquad$ = 0 is called Charac polynomial of A

Eigen Value :

$\qquad$ The <u>roots</u> of the Characteristic equation

of the matrix are called. Eigen Value or

Characteristic value.