



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



DEPARTMENT OF MATHEMATICS

UNIT II

ORTHOGONAL TRANSFORMATION OF A REAL SYMMETRIC MATRIX

UNIT - II

ORTHOGONAL TRANSFORMATION OF REAL SYMMETRIC MATRIX

Diagonalization of a real symmetric matrix:

Transforming a real symmetric matrix A into D by means of the transformation $N^T A N = D$ is known as orthogonal transformation. Here D is the diagonal matrix and N is the matrix whose columns are the normalized eigen vectors of A .

Problems:

- ① Diagonalize the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ by means of an orthogonal transformation?

Soln:

$$\text{Let } A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Step 1: To find the characteristic equation:

$$\lambda^3 - c_1 \lambda^2 + c_2 \lambda - c_3 = 0 \rightarrow \textcircled{1}$$

$$c_1 = 8 + 7 + 3$$

$$= 18$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



DEPARTMENT OF MATHEMATICS

$$C_2 = \begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} + \begin{vmatrix} 8 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix}$$
$$= (21 - 16) + (24 - 4) + (56 - 36)$$
$$= 45$$

$$C_3 = \begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{vmatrix}$$
$$= 8(21 - 16) + 6(-18 + 8) + 2(24 - 14)$$
$$= 0$$

Subs C_1, C_2, C_3 in ①,

$$\lambda^3 - 18\lambda^2 + 45\lambda = 0$$

Step 2: To find the eigen values:

$$\lambda^3 - 18\lambda^2 + 45\lambda = 0$$

$$\lambda(\lambda^2 - 18\lambda + 45) = 0$$

$$\lambda = 0, \lambda^2 - 18\lambda + 45 = 0$$

$$\lambda = 3, 15$$

$$\lambda = 0, 3, 15$$

Step 3: To find the eigen vectors:

$$(A - \lambda I)X = 0$$

$$\begin{pmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow \textcircled{2}$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



DEPARTMENT OF MATHEMATICS

Case (i) : $\lambda = 0$

$$\textcircled{2} \Rightarrow \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Taking first two rows,

$$\begin{array}{cccc} 8 & -6 & 2 & 8 \\ -6 & 7 & -4 & -6 \\ \hline & x_3 & x_1 & x_2 \end{array}$$

$$\frac{x_1}{\begin{vmatrix} -6 & 2 \\ 7 & -4 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 2 & 8 \\ -4 & -6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix}}$$

$$\frac{x_1}{10} = \frac{x_2}{20} = \frac{x_3}{26}$$

$$x_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

Case (ii) : $\lambda = 3$

$$\textcircled{2} \Rightarrow \begin{pmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Taking first two rows,

$$\begin{array}{cccc} 5 & -6 & 2 & 5 \\ -6 & 4 & -4 & -6 \\ \hline & x_3 & x_1 & x_2 \end{array}$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



DEPARTMENT OF MATHEMATICS

$$\frac{x_1}{-6 \ 2} = \frac{x_2}{4 \ -4} = \frac{x_3}{2 \ 5}$$

$$\frac{x_1}{16} = \frac{x_2}{8} = \frac{x_3}{-16}$$

$$\frac{x_1}{2} = \frac{x_2}{1} = \frac{x_3}{-2}$$
 ∴ The eigen vector is $x_2 = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$.

Case (iii): $\lambda = 15$

$$\begin{pmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$
 Taking first 2 rows,

$$\begin{matrix} -7 & -6 & 2 & -7 \\ -6 & -8 & -4 & -6 \end{matrix}$$

$$\frac{x_1}{-6 \ 2} = \frac{x_2}{-8 \ -4} = \frac{x_3}{2 \ -7}$$

$$\frac{x_1}{40} = \frac{x_2}{-40} = \frac{x_3}{20}$$
 ∴ The eigen vector is $x_3 = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$

Hence the modal matrix,

$$M = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



DEPARTMENT OF MATHEMATICS

Step 4: To find the normalised matrix N:

Normalising each column vector of M.

Dividing each element of first column by 3, second column by 3 & third column by 3, we get the normalized matrix N.

$$N = \begin{pmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$$

Step 5: Calculate $N^T A N$:

$$N^T = \frac{1}{3} \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$$

$$N^T A N = \frac{1}{3} \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix} \begin{pmatrix} 9 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix} \cdot \frac{1}{3}$$

$$= \frac{1}{3} \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix} \begin{pmatrix} 9 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix} \cdot \frac{1}{3}$$

$$N^T A N = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{pmatrix} = D$$

The diagonal elements are the eigen values of A.