



DEPARTMENT OF MATHEMATICS

UNIT II

ORTHOGONAL TRANSFORMATION OF A REAL SYMMETRIC MATRIX

3 Write the quadratic form for the following

matrix (i) $\begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 6 & -2 \\ 6 & 1 & -4 \\ -2 & -4 & -3 \end{bmatrix}$

Soln:

(i) $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 3 \end{pmatrix}$

$$Q = a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + 2a_{12}x_1x_2 + 2a_{23}x_2x_3 + 2a_{31}x_3x_1$$

$$Q = x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_2x_3 - 2x_3x_1$$

(ii) $Q = 2x_1^2 + x_2^2 - 3x_3^2 + 12x_1x_2 - 8x_2x_3 - 4x_1x_3$

Canonical form:

For a real quadratic form $Q = x^T A x$,

the canonical form is $y^T D y$ (or)

$$\lambda_1 y_1^2 + \lambda_2 y_2^2 + \dots + \lambda_n y_n^2$$



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Problems :

- ① Determine the nature of the following quadratic form $x_1^2 + 2x_2^2$

Soln:

Matrix of the quadratic form

$$= \begin{pmatrix} \text{Coef of } x_1^2 & \frac{1}{2} \text{ coef of } x_1 x_2 & \frac{1}{2} \text{ coef of } x_1 x_3 \\ \frac{1}{2} \text{ coef of } x_2 x_1 & \text{coef of } x_2^2 & \frac{1}{2} \text{ coef of } x_2 x_3 \\ \frac{1}{2} \text{ coef of } x_3 x_1 & \frac{1}{2} \text{ coef of } x_3 x_2 & \text{coef of } x_3^2 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The eigen values are 1, 2, 0

Nature = Positive Semidefinite

- ② Find the rank, index, signature and nature for the following canonical forms :

(i) $y_1^2 + 3y_2^2 + 4y_3^2$

(ii) $-y_1^2 + y_2^2 + 4y_3^2$

(iii) $3y_2^2 + 15y_3^2$

Soln:

(i) $y_1^2 + 3y_2^2 + 4y_3^2$

Rank = number of non-zero eigen values
= 3



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Index = number of positive square terms = 3
Signature = Difference between positive and negative square terms
 $= 3 - 0 = 3$
Nature = positive definite

(ii) $-y_1^2 + y_2^2 + 4y_3^2$

rank = 3

Index = 2

Signature = $2 - 1 = 1$

Nature = Indefinite

(iii) $3y_2^2 + 15y_3^2$

$= 0y_1^2 + 3y_2^2 + 15y_3^2$

rank = 2

Index = 2

Signature = 2

Nature = positive Semi definite



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$$\textcircled{1} \quad 2x^2 + 3y^2 + 2z^2 + 2xy$$

$$= \begin{matrix} & x & y & z \\ \begin{matrix} x \\ y \\ z \end{matrix} & \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix} \end{matrix}$$

$$\textcircled{2} \quad 2x_1^2 + x_2^2 - 3x_3^2 + 12x_1x_2 - 8x_2x_3 - 4x_1x_3$$

$$\begin{matrix} & x_1 & x_2 & x_3 \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} & \begin{bmatrix} 2 & 6 & -2 \\ 6 & 1 & -4 \\ -2 & -4 & -3 \end{bmatrix} \end{matrix}$$

$$\begin{bmatrix} 2 & 6 & -2 \\ 6 & 1 & -4 \\ -2 & -4 & -3 \end{bmatrix}$$