Bending ofbeams

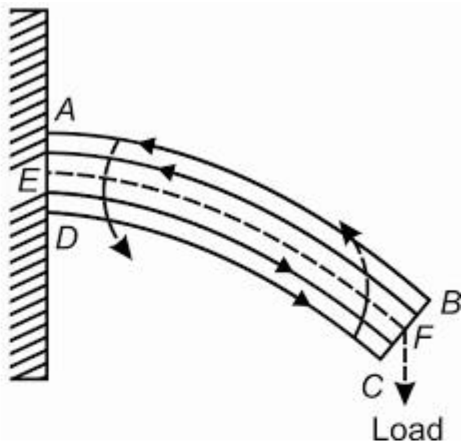
Abeamisdefinedasarod(or)barofuniformcrosssectionwhoselengthis very much greater than its other dimensions, such as breadth and thickness.

It is commonly used in the construction of bridges to support roofs of the buildingsetc.Sincethelengthofthebeamismuchgreaterthanitsotherdimensions the shearing stresses are very small.

Assumptions:

- Thelengthofthebeamshouldbelargecomparedtootherdimensions.
- Theappliedloadshouldbelargecomparedtotheweightofthebeam.
- Thecrosssectionofthebeamremainsconstantandhencethegeometrical moment of inertia also remains constant.
- Theshearingstressesare negligible.

ConsiderabeamABCD,whichismadeupofalargenumberofthinplanelayersare placedoneabovetheother.OneendofthebeamADisfixedintherigidsupportanda Loadisappliedin theother endBC.



TakingthelongitudinalsectionABCDofthebentbeam,thelayersintheupper half are elongated while those in the lower half are compressed.

In the middle there is a layer (MN) which is not elongated or compressed due tobendingofthebeam.Thislayeriscalledthe ***'neutralsurface'***andtheline(MN)at which the neutral layer intersects the plane of bending is called the ***'neutral axis'***.

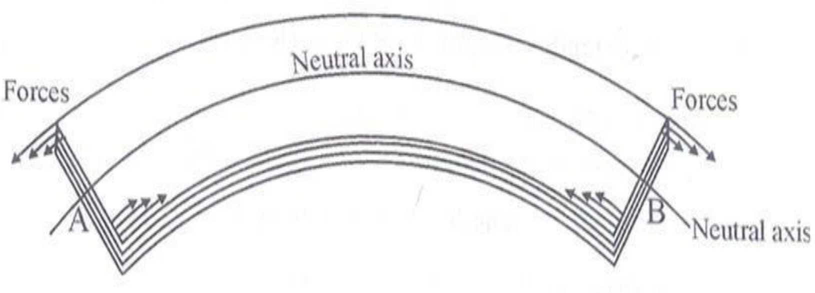
ThelayersbelowMNarecompressedandthoseaboveMNareelongatedand there will be such pairs of layers one above MN and one below MN experiencing same forces of elongation and compression due to bending and each pair forms a couple. *The resultant moments of all these internal couples are called the* ***internal bending moment***andintheequilibriumcondition,thisisequaltotheexternalbendingmoment

INTERNALBENDINGMOMENTOFTHEBEAM

When a beam bent, the restoring couple arises. This couple balances the externalcoupleduetoexternalloadiscalled **internalbendingmomentofthebeam**. At equilibrium,

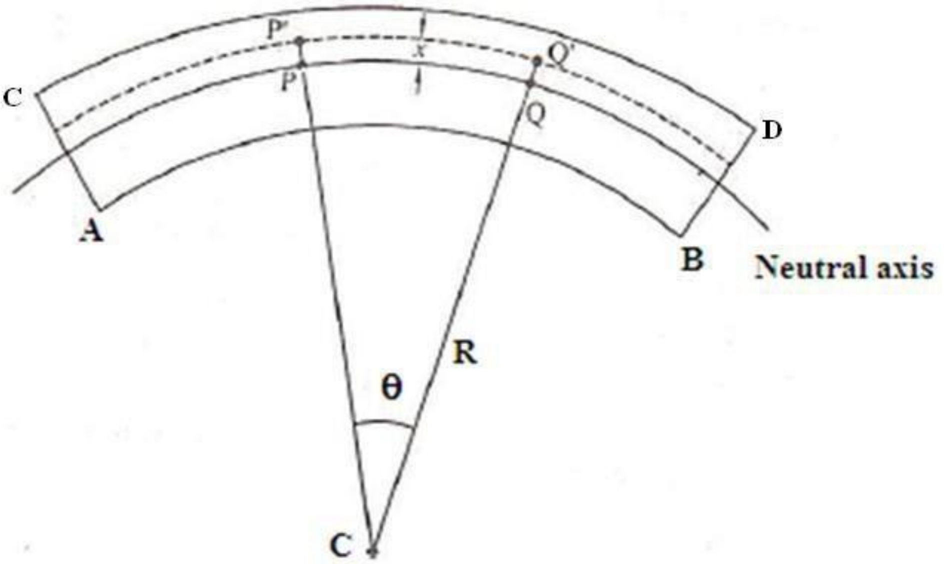
Restoring couple=Bending couple

A beam may be assumed to consist of a number of parallel longitudinal metallic fibers placed one over the other and are called as filaments. Let the beam be subjected to deforming forces at its ends, due to which it bends. Let us consider a filamentABatthecenterofthebeam.Itisfoundthatthefilaments(layers)lyingabove AB gets elongated, while the filaments lying below AB gets compressed.



Bending ofbeam.

Therefore the filament i.e. layer AB which remains unaltered taken as the referenceaxiscalledasNeutralaxisandtheplaneiscalledasneutralplane.Further,the deformation of any filament can be measured with reference to the neutral axis. The moment of couple due to elastic reactions (restoring couple) which balances the bending couple due to applied load is called the bending moment. Let us consider a beam under the action of deforming forces. The beam bends into a circular arc.



LetPQbetheneutralaxisofthebeamandP'Q'beanotherfilamentatdistanceyfrom PQ.IfRistheradiusofcurvatureoftheneutralaxisand θ istheanglesubtendedbyitat its centre of curvature 'C'.

Thenwecanwriteoriginallength**P**Q=Radiusx Angle

$$= R\theta \text{ ....} (1)$$

If $R x$ Is theradiusofcurvatureof thefilamentP'Q'.

$$P'Q' = (R\ x)\theta \text{ ....} (2)$$

ExtensionProducedinthefilamentP'Q'duetobending=P'Q'-PQ

$$= (Rx)\theta - R\theta$$

$$= x\theta \text{ ....} (3)$$

Longitudinalstrain=

Changeinlength xx
Original lengthR R

The Young's modulus of the filament P'Q'

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

Longitudinal stress on the filament P'Q' = Y longitudinal strain

$$= Y\frac{x}{R}$$

If A is the area of cross-section of the filament, then the tensile force on the filament.

= Longitudinal stress area

$$= Y\frac{x}{R}A$$

We know, moment of longitudinal force about the neutral axis = tensile force distance

$$Y\frac{x}{R}Ax\ \frac{Y}{R}Ax^2$$

Moment of all the forces about the neutral axis $= \frac{Y}{R}\sum Ax^2 \frac{Y}{R} I_g$

I_g is the geometrical moment of inertia and it is represented as AK^2

Where A is the area of the cross-section and K is radius of gyration. In equilibrium,

Bending moment of the beam = Moment of force

Internal bending moment of the beam $\frac{Y I_g}{R}$ (4)

SPECIAL CASES

Rectangular cross section

If b and d are the breadth and thickness of the beam, then $A\ bd$ and $K\ \frac{d^2}{12}$

$$I_g AK^2 \frac{bd^3}{12}$$

Using the value of $I_g\ \frac{bd^3}{12}$ in equation (4)

Internalbendingmomentoftherectangularbeam $= \frac{bd^3Y}{12R}$ .... (5)

Circularcrosssection

Ifrbetheradiusofthebeam,then Ar^2 and $K\frac{r^2}{4}$

$$I_g AK^2 \frac{r^4}{4}$$

Usingthevalue

$$I_g \frac{r^4}{4}$$ inequation(4)

Internalbendingmomentofthecircularbeam $\frac{r4Y}{4R}$ ……………(6)

DEPRESSIONOFACANTILEVER

DEFINITION

Alightbeamclampedhorizontallyatoneendandloadedwithaweight$W=Mg$ at the free end is called a **cantilever**.

Inequilibrium,

Externalbendingmoment =Internalbendingmoment

THEORY

Let us considerabeamfixedatone endand loadedat itsother freeendas shown in fig1.7.2.2.LetABistheneutralaxisofacantilever(abeamorrod)oflength‘l’isfixed at the end A and loaded at the free end B by a weight W. Due to load applied the cantilever is depressed to B’.

LetBB’representstheverticaldepressionatthefreeend.

Duetotheloadappliedatthefreeend,acoupleiscreatedbetweenthetwoforces.(i.e)

(i) Force(load‘W’)appliedatthefreeendtowardsdownwarddirectionand

(ii) Reaction(R)actingintheupwarddirectionatthesupportingend.

Thisexternalbending coupletendsto bend thebeamin theclockwisedirection. But,sinceoneendofthebeamisfixed,thebeamcannotrotate.Thereforetheexternal bendingcouplemustbebalancedbyanotherequalandoppositecouple,createddueto the elastic nature of the body called as internal bending moment.

ConsiderthesectionofthecantileverPatadistance'x'fromthefixedendA.Qis another point at a distance dx from P i.e., PQ = dx. It is at a distance (l-x) from the loaded end B'. Considering the equilibrium of the portion PB', there is a force of reaction W of P. Let O be the centre of curvature and R be the radius of curvature.

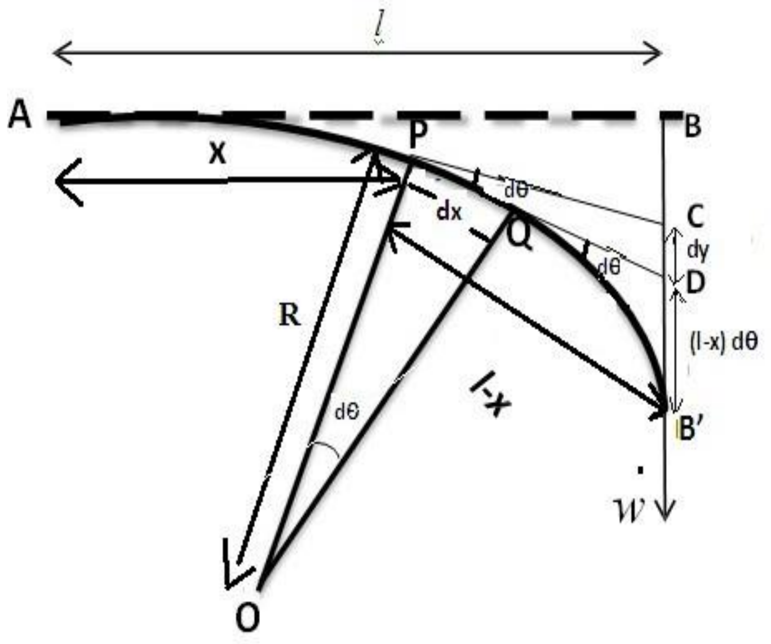


Fig1.7.2.2.Depressionofacantilever.

• Theexternalbendingmoment=*WPB'*=W(l-x) .... (1)

• Internalbendingmomentofthecantilever $\frac{YI_g}{R}$ .... (2)

R-Radiusofthecurvatureoftheneutralaxisat P.

Where Y – Young's modulus of the cantilever.

I_g - Geometrical moment of inertia of its cross-section.

In the equilibrium position,

External bending moment = Internal bending moment

$$W(l-x) = \frac{YI_g}{R}$$

$$R = \frac{YI_g}{W(l-x)} \qquad \ldots\ldots\ldots\ldots\ldots(3)$$

Let Q be another point on the bent cantilever at small distance 'dx' from P. Since P and Q are very near, we can assume that the radius of curvature R is practically the same.

The tangents are drawn at P and Q meeting the vertical line BB' at C and D. Let d□ be the angle between the tangents at P and Q.

$$Sin\, d\theta \approx d\theta = \frac{dx}{R}$$

$$R = \frac{dx}{d\theta} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Substituting the value of R from (3) in (4), we have

$$\frac{dx}{d\theta} = \frac{YI_g}{W(l-x)}$$

$$d\theta = \frac{W(l-x)dx}{YI_g} \qquad \ldots\ldots\ldots\ldots\ldots\ldots..(5)$$

If 'dy' is the depression due to the curvature at PQ

$$dy = (l-x)d\theta \qquad \ldots\ldots\ldots\ldots\ldots\ldots.(6)$$

Substituting value of d□

$$dy = (l-x)\frac{W(l-x)}{YI_g}dx$$

$$dy = \frac{W(l-x)^2}{YI_g}dx \qquad \ldots\ldots\ldots\ldots\ldots\ldots.(7)$$

R-Radiusofthecurvatureoftheneutralaxisat P.

Tofindthetotaldepressionatthefreeendofthecantileverequation(7)hastobeintegrated from 0 to l.

$$dyW \int_0^l \frac{(lx)^2}{YI_g} dx$$

$$y = \frac{W}{YI_g}\int_0^l (l-x)^2\,dx = \frac{W}{YI_g}\int_0^l (l^2 + x^2 - 2lx)\,dx$$

$$y = \frac{W}{YI_g}\left[l^2 x + \frac{x^3}{3} - \frac{2l x^2}{2}\right]_0^l$$

$$y = \frac{W}{YI_g}\left[l^3 + \frac{l^3}{3} - l^3\right]$$

$$y = \frac{Wl^3}{3YI_g} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

Equation (8) gives the depression produced by the cantilever at the free end.

SPECIAL CASES

Rectangular cross section

If b and d are the breadth and thickness of the beam, then $A = bd$ and $K^2 = \frac{d^2}{12}$

Substituting the values of $I_g = AK^2$, $I_g = \frac{bd^3}{12}$ and $W = Mg$ in equation (8)

$$\text{Depression } y = \frac{4Mgl^3}{Ybd^3} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(9)$$

Circular cross section

If r be the radius of the beam, then $A = \pi r^2$ and $K^2 = \frac{r^2}{4}$

Substituting the values of $I_g = AK^2 = \frac{\pi r^4}{4}$ and $W = Mg$ in equation (8)

$$\text{Depression } y = \frac{4Mgl^3}{3Y\pi r^4} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

EXPERIMENTAL DETERMINATION OF YOUNG'S MODULUS BY CANTILEVER DEPRESSION METHOD

CONSTRUCTION

Oneendofthebeamisrigidlyclampedatoneendtotheedgeofthetable using G-clamp. A tall pin P is fixed vertically to the free end of the bar. A loop ofcottonstringorahookisattachedtothisendofthebarandaweighthangeris suspendedfromit.Atravellingmicroscopeisfocusedonthetipofthepinasshownin fig.

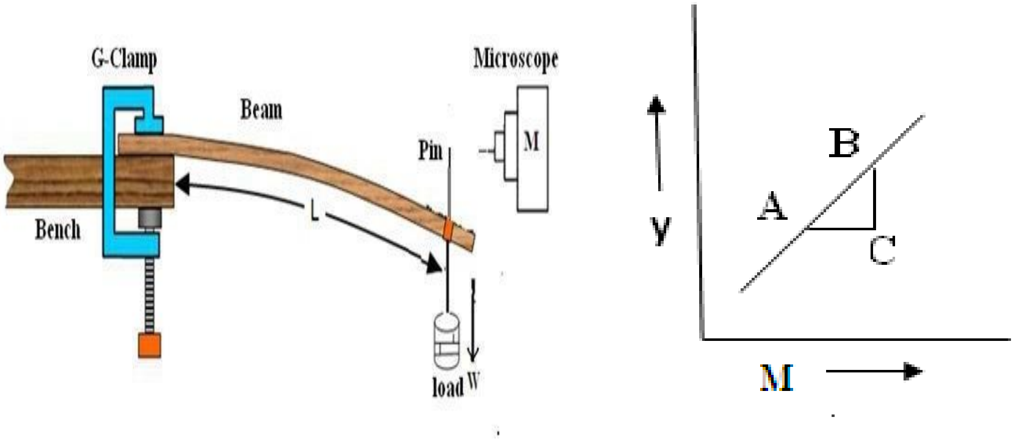


Fig1.7.2.3Experientalverificationofacantilever.

PROCEDURE

Adeadloadwithoutanyslottedweightsisattachedtothehook.Themicroscope isadjustedsuchthatthehorizontalcrosswirecoincideswiththetipoftheimageofthe pinandthereadingontheverticalscaleistaken.Loadsareaddedtothehangerinstepsof 50g and every time, the readings are noted on the vertical scale. A travelling microscopeisfocusedonthetipofthepinasshowninfig.Theseobservationsarealso repeated while unloading in the same. Steps and the readings are tabulated. The mean depression ‘y’ for a load ‘M’ kg is found fromthe tabulated readings. Theobservationsaretabulatedasfollows.

Graphicalmethod

A graph is drawn between the load (M) along X-axis and elevation (y) along Y-axis.Itisfoundtobeastraightlineasshowninfig.Theslopeofthestraightlinegives the value of (y/M).

Load $10^3 kg$	Microscopereadings							depression (y)for100g $10^2 m$	Mean (M/y) Kgm^{-1}
	Loading			Unloading			Mean		
	MSRcm	VSR cm	TR cm	MSRcm	VSR Cm	TR Cm	$10^2 m$		
W									
W+50									
W+100									
W+150									
W+200									

HenceYoung'smodulusofthecantilevercanbecalculatedas

$$Y = \frac{4gl^3M}{bd^3} \quad \frac{\quad}{y}$$