SNS COLLEGE OF TECHNOLOGY

An Autonomous Institution

Coimbatore-35

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

23ECT202 – SIGNALS AND SYSTEMS

II YEAR/ III SEMESTER

UNIT 3 – LTI CONTINUOUS TIME SYSTEMS

TOPIC – **LTI SYSTEMS**

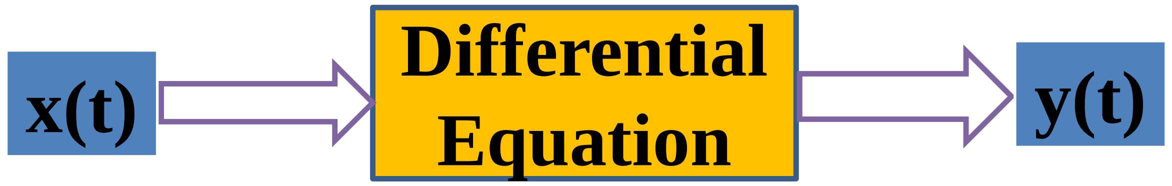
LTI SYSTEM

- Linear Time Invariant Systems (LTI) are characterized with the help of

1. Differential Equation
2. Impulse Response
3. Block Diagrams
4. State Variable description
5. Transfer Functions

DIFFERENTIAL EQUATION

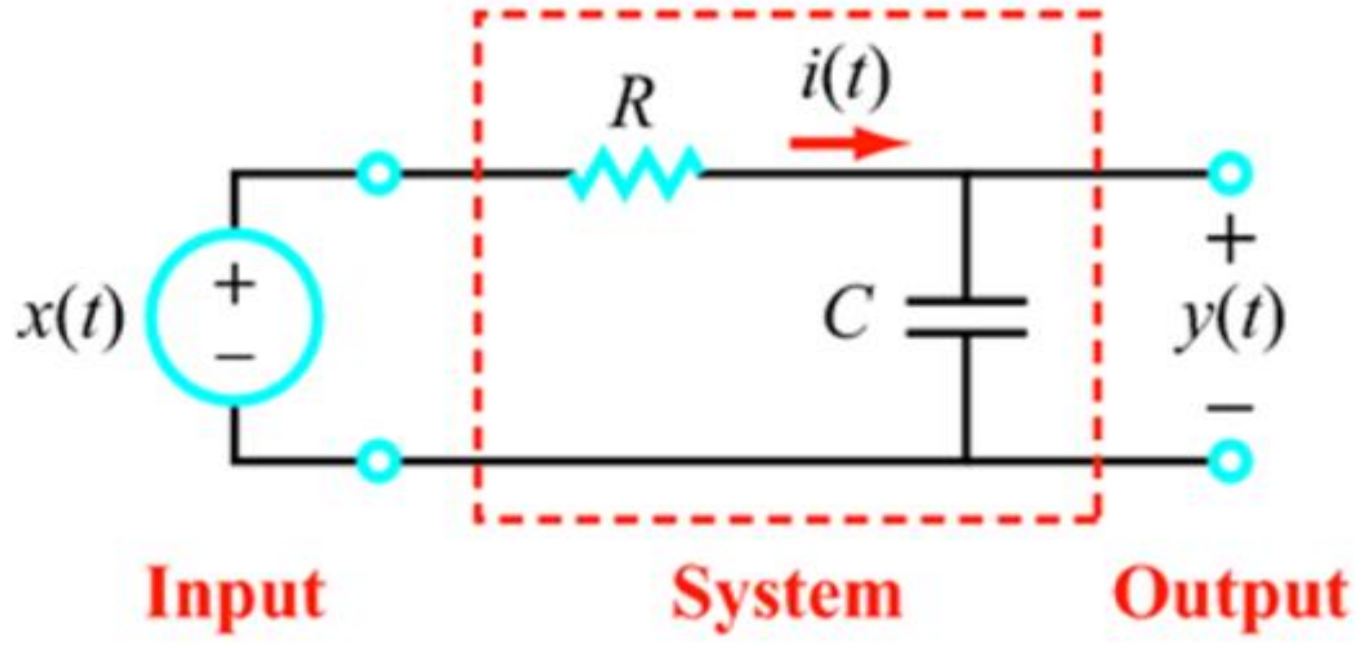
- It is used to represent continuous time linear time invariant system
- It relates the input and output of the system

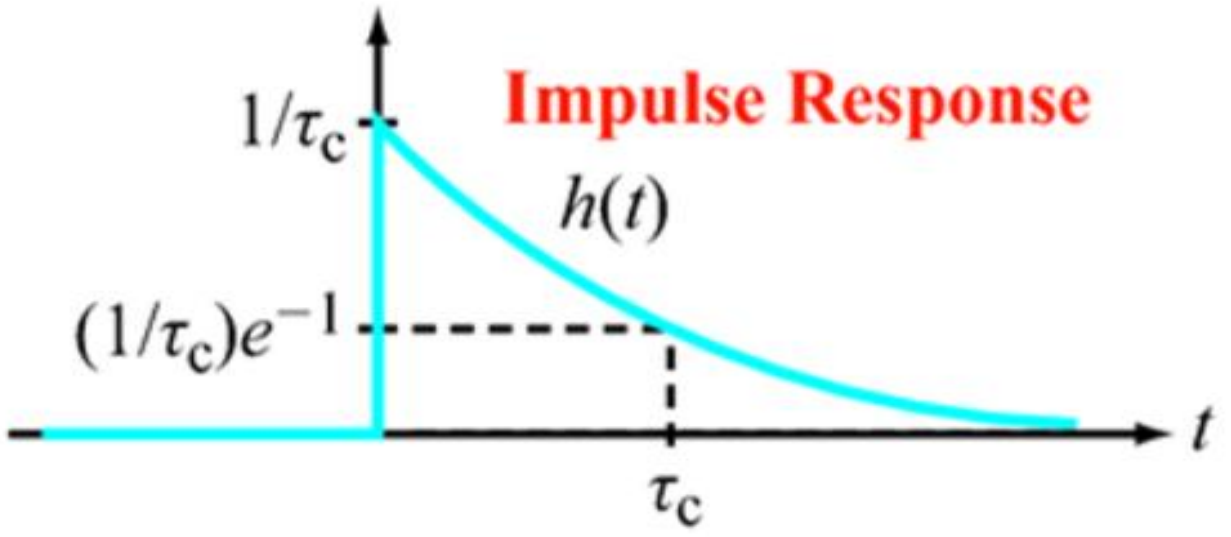


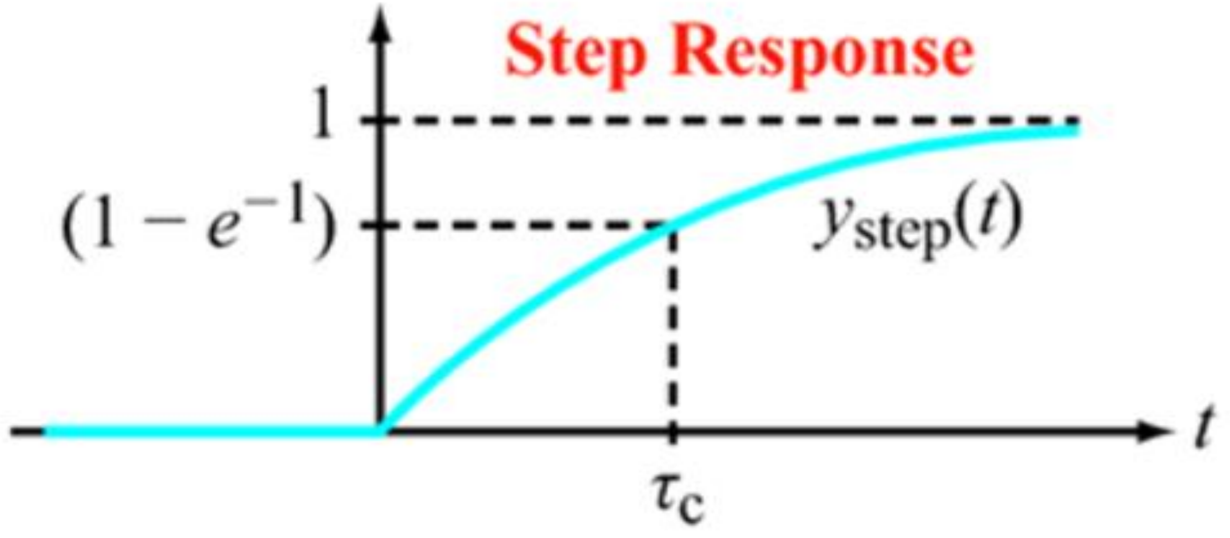
- Differential Equation has two Components

1. Natural Response
2. Forced Response

DIFFERENTIAL EQUATION







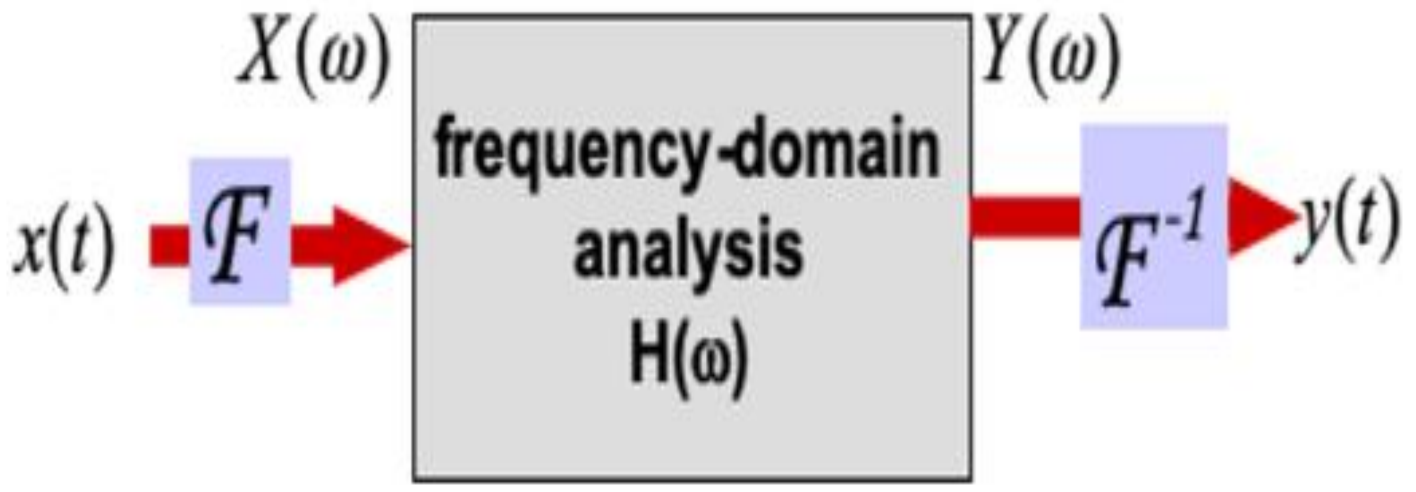
LTI SYSTEM

- **System Transfer Function:** Ratio of the output to the input.

$$H(s) = \frac{Y(s)}{X(s)}$$

- **Frequency Response:**

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$



LTI SYSTEM

- Condition for an Linear Time Invariant (LTI) system to be causal:

$$h(t) = 0, t<0$$

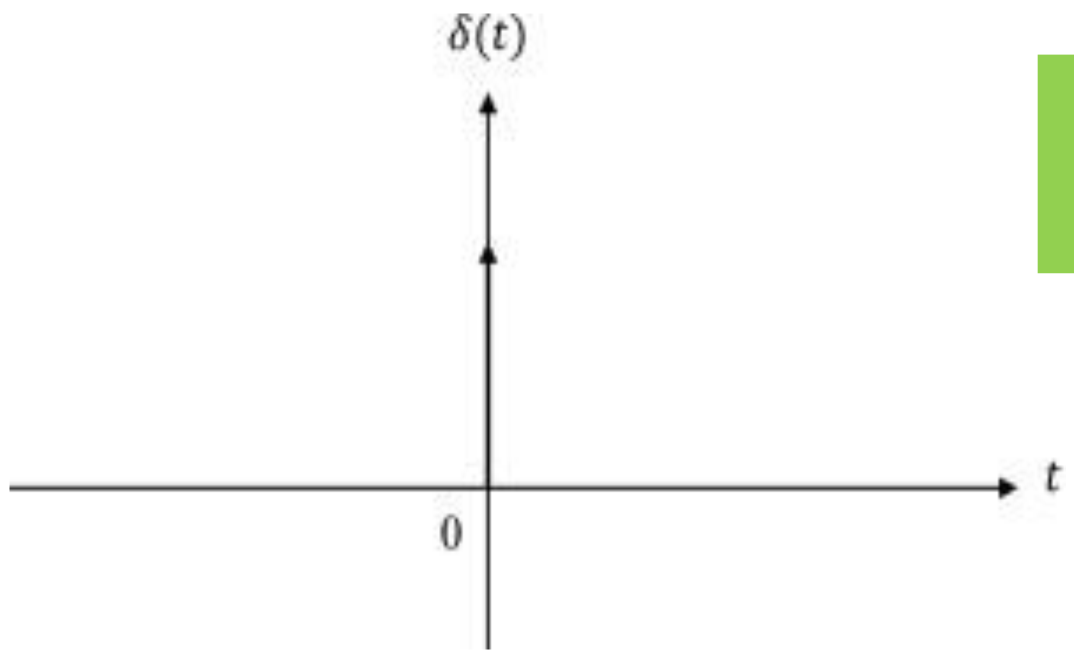
- Condition for an Linear Time Invariant (LTI) system to be stable:

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

IMPULSE RESPONSE

- Impulse response is the output generated by the system, when an unit impulse is applied at the input.

$x(t) = \delta(t)$ → **LTI System** → $y(t) = h(t)$

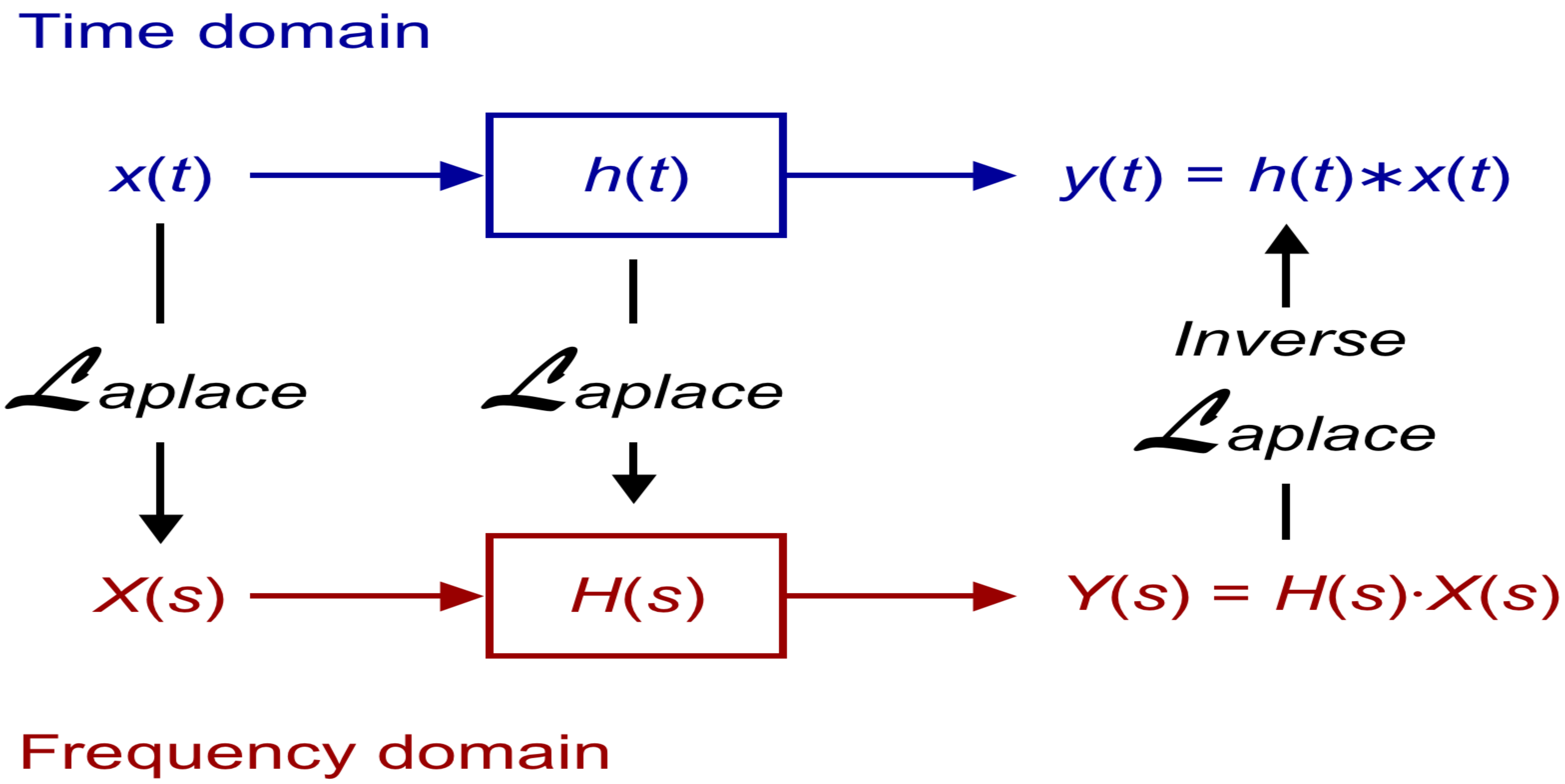


$\delta(t) = 1$ for $t = 0$
$= 0$ for $t \neq 0$

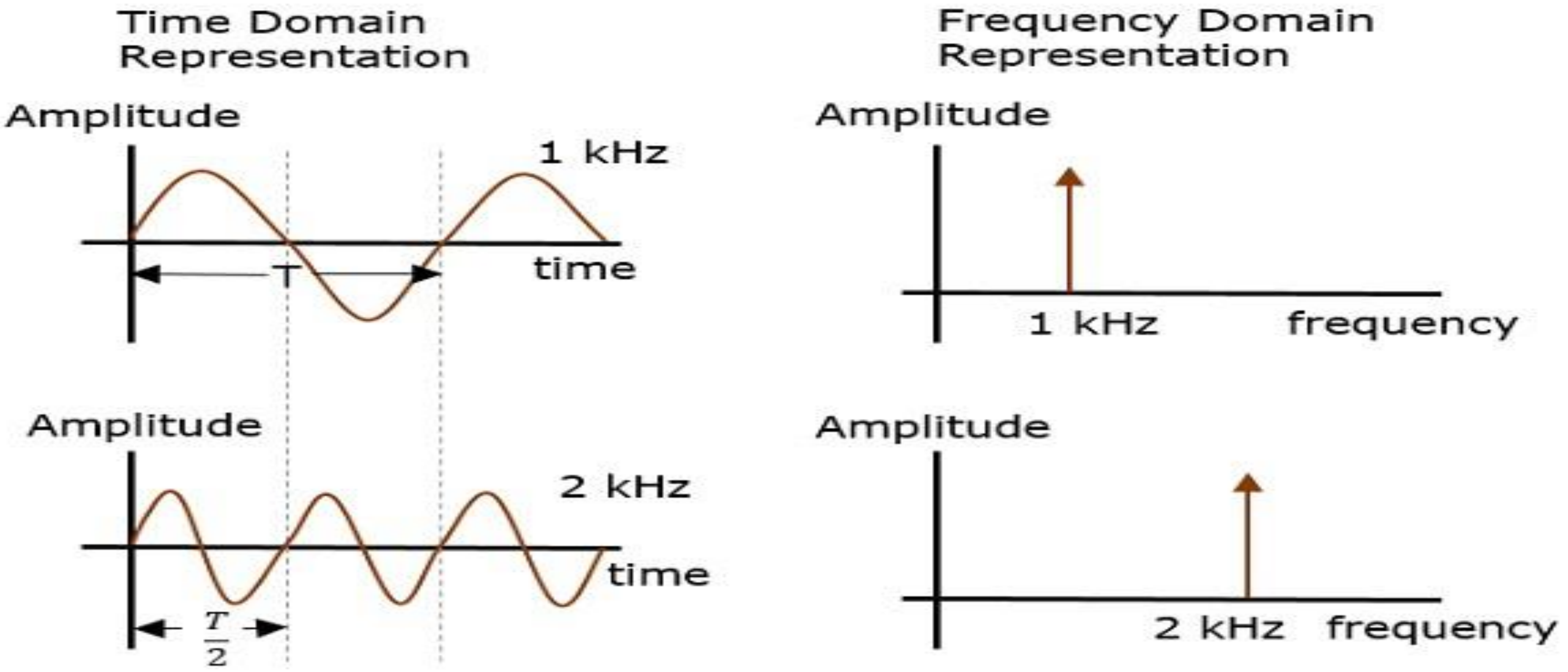
- $H(s) = \dfrac{Y(s)}{X(s)}$

- $h(t) = L^{-1}\{H(s)\}$

TIME DOMAIN INTO FREQUENCY DOMAIN



TIME DOMAIN INTO FREQUENCY DOMAIN



TO FIND IMPULSE RESPONSE

Differential Equation

⇩

Applying Laplace Transform

⇩

Algebraic Equation

⇩

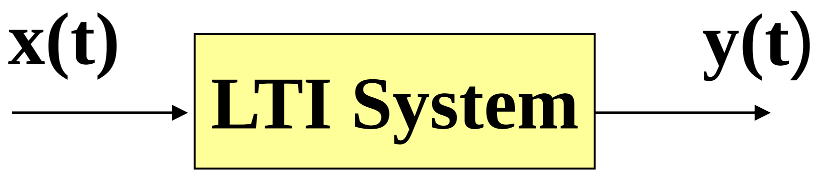
Solving for roots

⇩

Inverse Laplace Transform

CONVOLUTION INTEGRAL

- Any input can be expressed using the unit impulse function



$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\tau$$

CONVOLUTION INTEGRAL - EXAMPLE

- Consider a CT-LTI system. Assume the impulse response of the system is h(t)=e^(-at) for all a>0 and t>0 and input x(t)=u(t). Find the output.

u(t) → h(t)=e^-at → y(t)

$$y(t) = h(t) * x(t) = h(t) * u(t)$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)u(t-\tau)d\tau$$

$$= \int_{-\infty}^{\infty} \left(e^{-a\tau} \cdot u(\tau)\right)u(t-\tau)d\tau$$

Because t>0

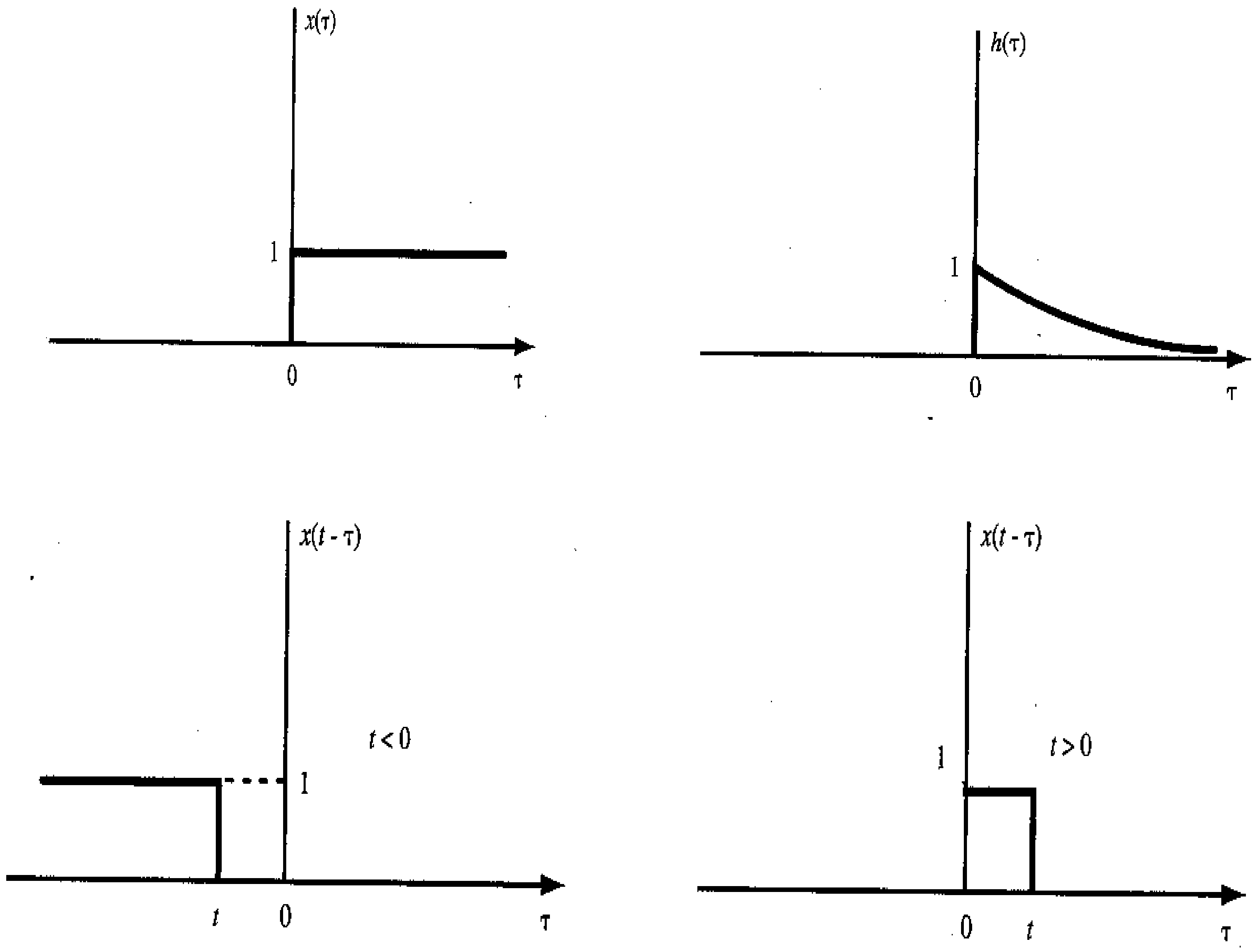
$$\int_{0}^{t} \left(e^{-a\tau}\right)d\tau = \frac{1}{-a}(e^{-at}-1)$$

$$= \frac{1}{a}(1-e^{-at})u(t)$$

$$y(t) = h(t) * x(t) = h(t) * u(t)$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

CONVOLUTION INTEGRAL - REPRESENTATION



$$y(t) = h(t) * x(t) = h(t) * u(t)$$

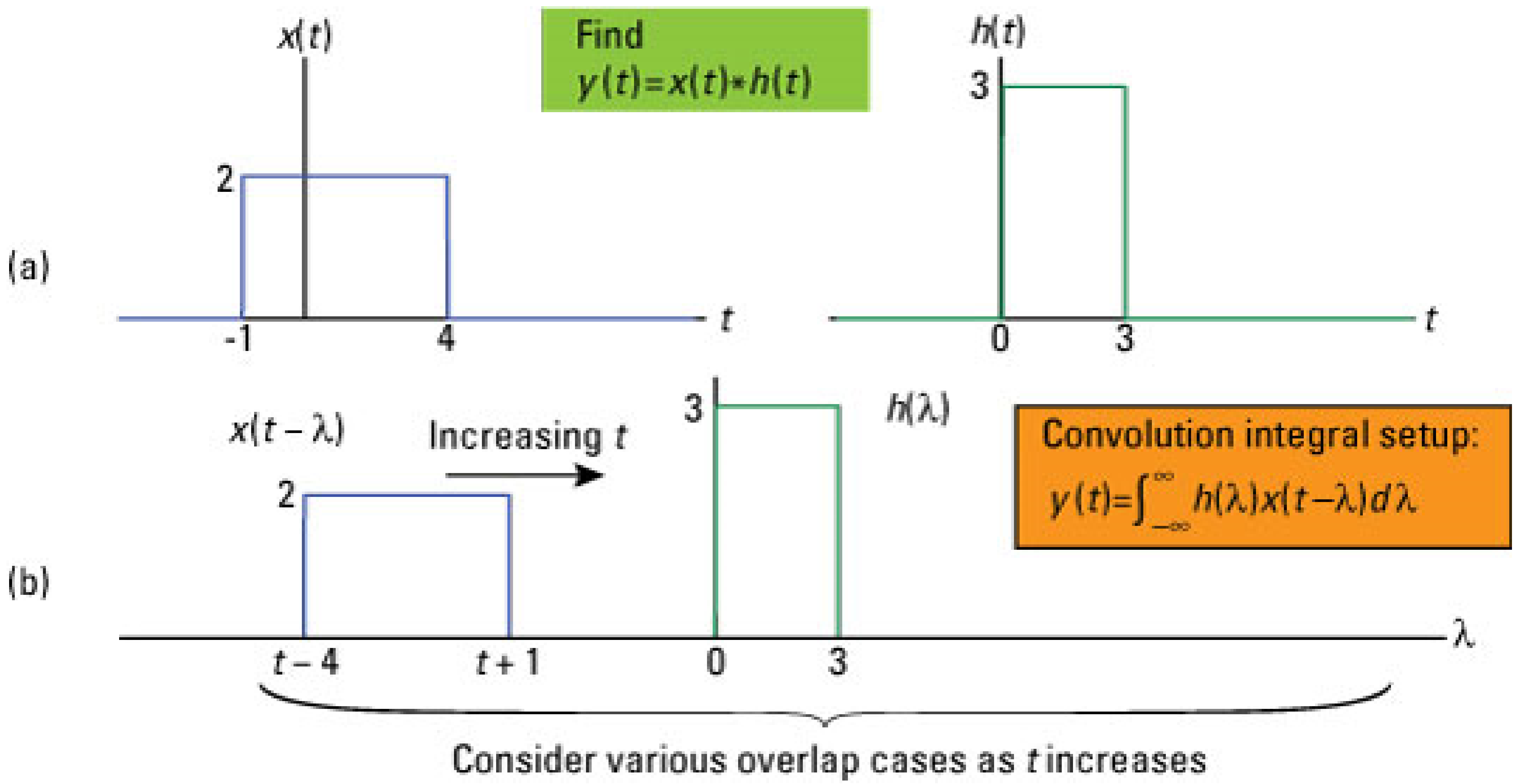
$$y(t) = \int_{-\infty}^{\infty} h(\tau) u(t-\tau) d\tau$$

$$= \int_{-\infty}^{\infty} \left(e^{-a\tau} \cdot u(\tau)\right) u(t-\tau) d\tau$$

$$\int_{0}^{t} \left(e^{-a\tau}\right) d\tau = \frac{1}{-a}(e^{-at} - 1)$$

$$= \frac{1}{a}(1 - e^{-at}) u(t)$$

CONVOLUTION INTEGRAL



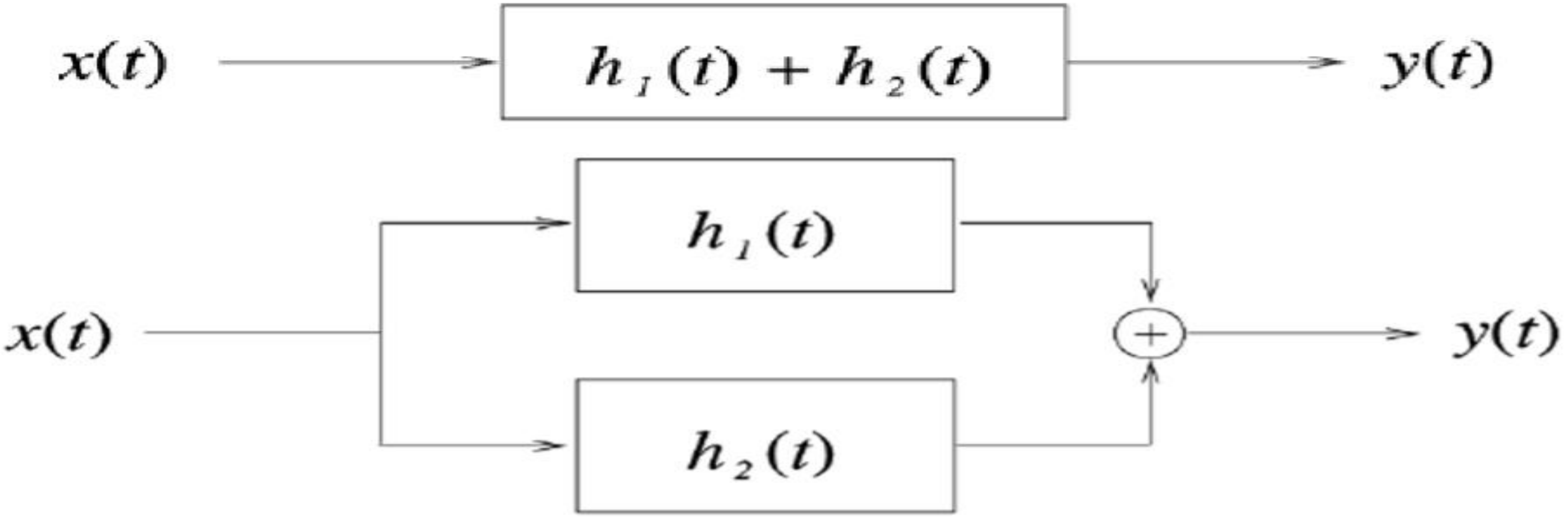
PROPERTIES OF CONVOLUTION INTEGRAL

COMMUTATIVE

$$x(t) * h(t) = h(t) * x(t)$$

DISTRIBUTIVE

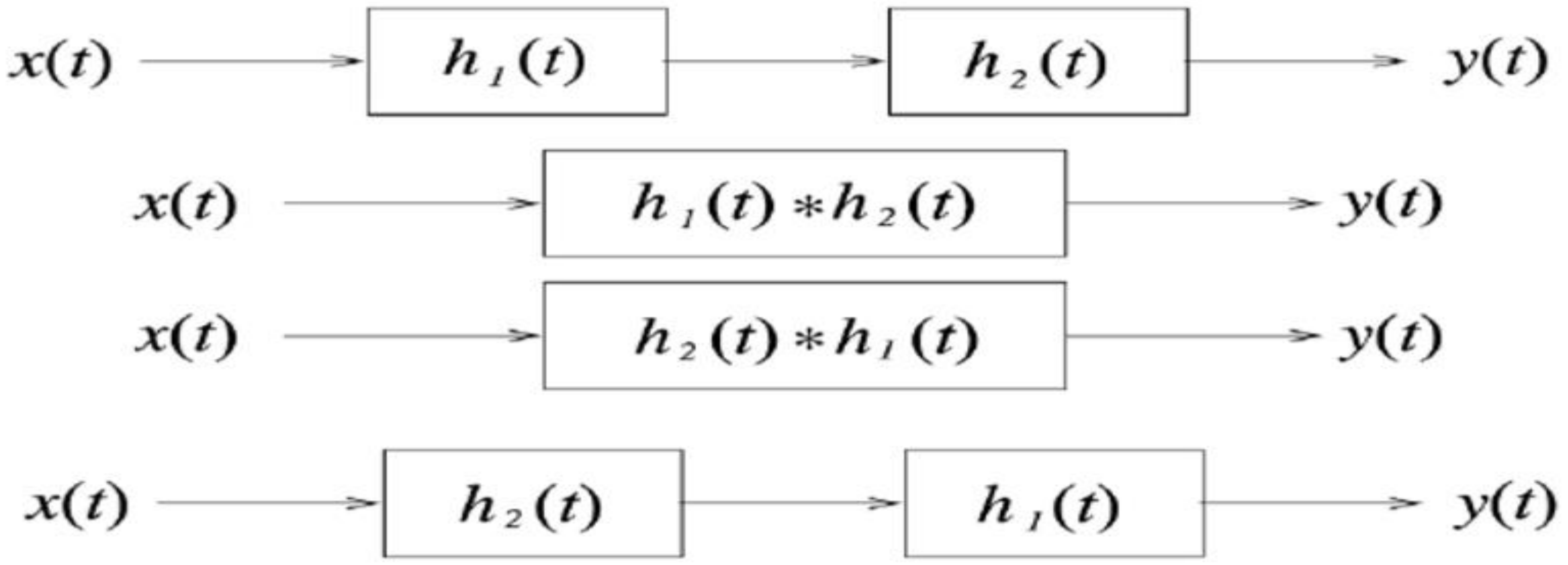
$$x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$$



ASSOCIATIVE PROPERTY

$$[x(t) * h_1(t)] * h_2(t) = x(t) * [h_1(t) * h_2(t)]$$

$$[x(t) * h_2(t)] * h_1(t) = x(t) * [h_2(t) * h_1(t)]$$



ASSESSMENT

1. Define LTI System.
2. The system transfer function is given by -------------
3. List the properties of convolution integral.
4. ------------- relates the input and output of the system.
5. What is meant by impulse response?
6. The condition of an LTI system to be causal is given by -------------
7. Associative property is defined as -----------
8. The condition of stability of an LTI system is ---------

THANK YOU