Calculation of Carrier Concentration at 0 K

The number of electrons per unit volume is called carrier concentration. It is calculated by summing up the product of the density of states Z(E) and Fermi distribution function F(E).

Carrier concentration $n_c = \int Z(E)\, F(E)\, dE$

Substituting Z(E) and F(E) in the above equation, we get,

$$n_c = \int \frac{4\pi}{h^3}(2m)^{3/2}E^{1/2}\frac{1}{1+e^{(E-E_F)/KT}}\,dE \quad \ldots (1)$$

For metals at T = 0 K, the upper most occupied level is E_F and F(E) = 1. Now the equation (1) becomes,

$$n_c = \int_0^{E_F} \frac{4\pi}{h^3}(2m)^{3/2}E^{1/2}\,dE$$

$$= \frac{4\pi}{h^3}(2m)^{3/2}\int_0^{E_F} E^{1/2}\,dE$$

$$n_c = \frac{4\pi}{h^3}(2m)^{3/2} * \left[\frac{E^{3/2}}{3/2}\right]_0^{E_F}$$

$$n_c = \frac{8\pi}{3h^3}(2mE_F)^{3/2} \quad \ldots\ldots\ldots..(2)$$

This equation is the carrier concentration or density of charge carrier at 0 K in terms of Fermi energy.

Calculation of Fermi Energy

Fermi energy is calculated from the expression of carrier concentration.

$$n_c = \frac{8\pi}{3h^3}(2mE_F)^{3/2}$$

$$(E_F)^{3/2} = \frac{3h^3 n_c}{8\pi(2m)^{3/2}}$$

Multiply the power of 2/3 on both sides of the above equation, we have

$$E_F = \left[\frac{3h^3 n_c}{8\pi(2m)^{\frac{3}{2}}}\right]^{\frac{2}{3}}$$

$$E_F = \left[\frac{3h^3 n_c}{\pi(8m)^{\frac{3}{2}}}\right]^{\frac{2}{3}} \qquad \left(\therefore\ (8m)^{\frac{3}{2}} = 8(2m)^{\frac{3}{2}}\right)$$

Rearrange the above equation, we get

$$E_F = \frac{h^2}{8m}\left(\frac{3n_c}{\pi}\right)^{2/3}$$

This is the expression for Fermi energy of electrons in solids at absolute zero temperature. It is depends only on the density of electrons of metals.