Weighted residual techniques

General Trial function:

$$u = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

Substitute the trial function in differential equation to get the residue function.

POINT COLLOCATION METHOD:

R(x)=0

R(x) ⟹ ***Residue function***

Here the point x is between these limits. If the limit is 0 to 1.

For two unknowns any two point between these limits. For example

X value is $\frac{1}{4}$ *X value is* $\frac{3}{4}$

For three unknowns any two point between these limits. For example

X value is $\frac{1}{4}$ *X value is* $\frac{2}{4}$ *X value is* $\frac{3}{4}$

SUB DOMAIN METHOD:

$$\int R(x).dx = 0$$

Here the limit should be taken into two intervals. If the limit is O to 1

The limit varies between 0 - 0.5 $\int_0^{0.5} R(x)dx = 0$

The limit varies between 0.5 - 1 $\int_{0.5}^{1} R(x)dx = 0$

LEAST SQUARE METHOD:

$$\int R(x).\frac{dR}{da_i}dx = 0 \qquad i = 0,1,2,3.............$$

Differentiate the R(x) with respect to a_1, a_2

GALERKIN METHOD:

$$\int R(x).\phi(x).dx = 0$$

$\phi(x)$ ⟹ *Weighting function (Function associated with unknown trial function*

$$\phi(x) = \phi_0 + a_2\phi_1 + a_3\phi_2$$

Rayleigh Ritz Method

For Beam the Fourier series equation $y = \sum_{n=1,2,3}^{\infty} a \sin\frac{n\pi x}{l}$

$$y = a_1 \sin\frac{\pi x}{l} + a_2 \sin\frac{3\pi x}{l}$$

For Cantilever Bar (Axial Loading)

$$u = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots\ldots\ldots\ldots + a_n x^n$$

Where a_1, a_2 & a_3 are Ritz parameters

THE TOTAL POTENTIAL ENERGY OF THE BEAM:

$$\pi = U - H$$

U ↳ *Strain Energy, H* ↳ *Work Done by External Force*

STRAIN ENERGY FOR TRANSVERSE LOADING:

$$U = \frac{EI}{2}\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx$$

STRAIN ENERGY FOR AXIAL LOADING:

$$U = \frac{EA}{2}\int_0^l \left(\frac{d^2u}{dx^2}\right)^2 dx$$

WORK DONE BY EXTERNAL FORCE:

$H = \int_0^l wydx$ For SSB with UDL throughout its length

$H = Wy_{max}$ For SSB with Point Load at its mid - point of length

$H = \int_0^1 wydx + Wy_{max}$ For SSB with UDL throughout its length and Point Load at its mid - point of length

$H = \int_0^1 Pdx = \int^l \rho Audx$ For Cantilever Bar with axial loading

BENDING MOMENT:

$$M = EI\frac{d^2y}{dx^2}$$

E → Young's 1nodulus, I → Moment of Inertia

EXACT SOLUTION:

SSB Max Deflection with UDL throughout its length $y_{\max} = \frac{5}{384}\frac{wl^4}{EI}$

SSB Max Bending Moment with UDL throughout its length $M_{centre} = \frac{wl^2}{8}$

SSB Max Deflection with Point Load at its mid - point of length $y_{\max} = \frac{wl^3}{48EI}$

SSB Max Bending Moment with Point Load at its mid - point of length $M_{centre} = \frac{wl}{4}$

SSB Max Deflection with UDL throughout its length and Point Load at its mid - point of length $y_{\max} = \frac{5}{384}\frac{wl^4}{EI} + \frac{wl^3}{48EI}$

SSB Max Bending Moment with UDL throughout its length and Point Load at its mid - point of length $M_{centre} = \frac{wl^2}{8} + \frac{wl}{4}$

GENERAL TRIGONOMETRIC FUNCTIONS USED IN RITZ METHOD:

$$\sin^2 x = \frac{1-\cos 2x}{2} \qquad \cos^2 x = \frac{1+\cos 2x}{2}$$

$$\sin A \sin B = \frac{\cos(A-B)+\cos(A+B)}{2}$$

SPRINGS: THE TOTAL POTENTIAL ENERGY OF THE BEAM:

$$\pi = U - H$$

U ↳ Strain Energy, H ↳ Work Done by External Force

STRAIN ENERGY: $U = \frac{1}{2} k\delta^2$ k-Stiffness of Spring, δ -Deflection

WORK DONE BY EXTERNAL FORCE: H=Fu, Force acting on the spring,
u- Displacement