



DEPARTMENT OF MATHEMATICS

23MAT101 - MATRICES AND CALCULUS

UNIT-III DIFFERENTIAL CALCULUS

Centre of Curvature

$$\bar{x} = x - \frac{y_1}{y_2} (1 + y_1^2)$$

$$\bar{y} = y + \frac{1}{y_2} (1 + y_1^2)$$

(\bar{x}, \bar{y}) is the coordinates of the centre of curvature:

Circle of curvature

The equation of the circle of curvature is

$$(x - \bar{x})^2 + (y - \bar{y})^2 = \frac{R^2}{2}$$

Problems:

① Find the centre of curvature of $y = x^2$ at the origin.

Soln:

$$y = x^2 \quad \text{point: } (0, 0)$$

$$\frac{dy}{dx} = y_1 = 2x \quad ; \quad y_1(0, 0) = 2(0) = 0$$

$$\frac{d^2y}{dx^2} = y_2 = 2 \quad ; \quad y_2(0, 0) = 2$$

The centre of curvature is,

$$\left. \begin{aligned} \bar{x} &= x - \frac{y_1}{y_2} (1 + y_1^2) \\ &= 0 - 0 (1 + 0) \\ \bar{x} &= 0 \end{aligned} \right\} \begin{aligned} \bar{y} &= y + \frac{1}{y_2} (1 + y_1^2) \\ &= 0 + \frac{1}{2} (1 + 0) \\ \bar{y} &= \frac{1}{2} \end{aligned}$$

$$\therefore (\bar{x}, \bar{y}) = (0, \frac{1}{2})$$



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$$y_2 = \frac{x(-y_1) - (-y)}{x^2} \quad (1)$$

$$= \frac{-xy_1 + y}{x^2}$$

$$y_2(3,4) = \frac{-3\left(\frac{-4}{3}\right) + 4}{3^2}$$

$$= \frac{4+4}{9}$$

$$y_2 = \frac{8}{9}$$

$$\bar{x} = x - \frac{y_1}{y_2} (1 + y_1^2)$$

$$= 3 - \frac{(-4/3)}{8/9} \left[1 + \left(-\frac{4}{3}\right)^2 \right]$$

$$= 3 + \frac{4}{3} \times \frac{9}{8} \left[1 + \frac{16}{9} \right]$$

$$= 3 + \frac{3}{2} \left(\frac{9+16}{9} \right)$$

$$= 3 + \frac{3}{2} \times \frac{25}{9}$$

$$= 3 + \frac{25}{6}$$

$$= \frac{18+25}{6}$$

$$\bar{x} = \frac{43}{6}$$



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$$\bar{y} = y + \frac{1}{y_2} (1 + y_1^2)$$

$$= 4 + \frac{1}{8/9} \left[1 + \left(-\frac{4}{3}\right)^2 \right]$$

$$= 4 + \frac{9}{8} \left[1 + \frac{16}{9} \right]$$

$$= 4 + \frac{9}{8} \left[\frac{9 + 16}{9} \right]$$

$$= 4 + \frac{9}{8} \times \frac{25}{9}$$

$$= 4 + \frac{25}{8}$$

$$= \frac{32 + 25}{8}$$

$$\bar{y} = \frac{57}{8}$$

$$P = \frac{(4 + y_1^2)^{3/2}}{8/9}$$

$$= \frac{\left[1 + \frac{16}{9} \right]^{3/2}}{8/9} = \frac{\left(\frac{25}{9}\right)^{3/2}}{8/9}$$

$$= \frac{\left[\frac{5^2}{3^2}\right]^{3/2}}{8/9} = \frac{5^3}{3^3} \times \frac{9}{8}$$

$$= \frac{125}{27} \times \frac{9}{8} = \frac{125}{24} \Rightarrow \boxed{P = \frac{125}{24}}$$



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$$\therefore \textcircled{1} \Rightarrow \left(x - \frac{43}{6}\right)^2 + \left(y - \frac{51}{8}\right)^2 = \left(\frac{125}{24}\right)^2.$$