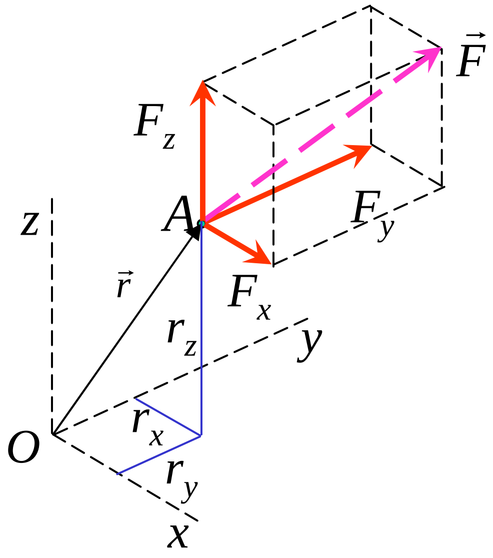
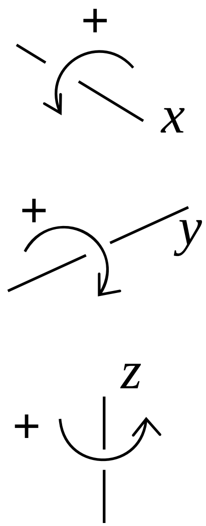
Moment and couple

In 3-D, because the determination of the distance can be tedious, a vector approach becomes advantageous.

$$\boxed{\vec{M}_o = \vec{r} \times \vec{F}} \quad \Rightarrow \quad \vec{M}_o = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$\vec{M}_o = (r_y F_z - r_z F_y)\hat{i} + (r_z F_x - r_x F_z)\hat{j} + (r_x F_y - r_y F_x)\hat{k}$$



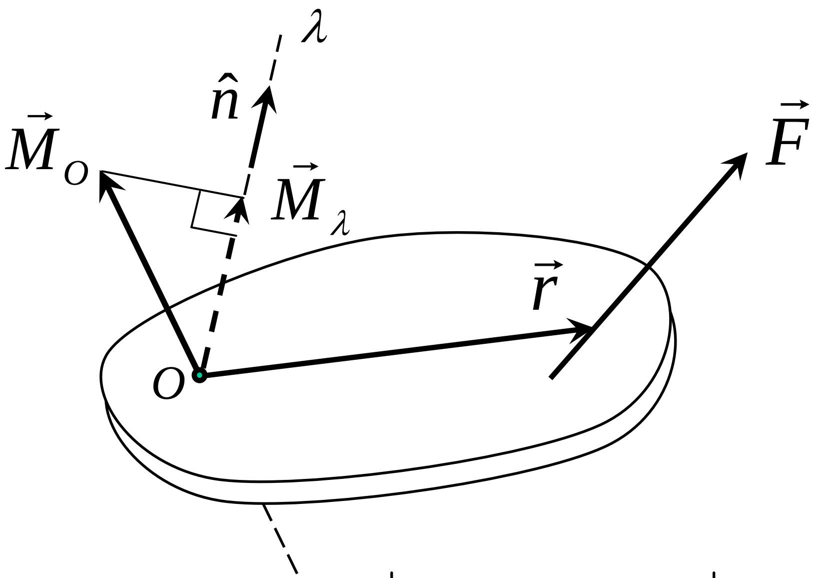


$$M_x = -F_y r_z + F_z r_y$$

$$M_y = F_x r_z - F_z r_x$$

$$M_z = -F_x r_y + F_y r_x$$

Moment about an arbitrary axis



Find moment about λ axis

1. Calculate moment

$$\vec{M}_o = \vec{r} \times \vec{F}$$

2. Calculate projection of moment on λ axis

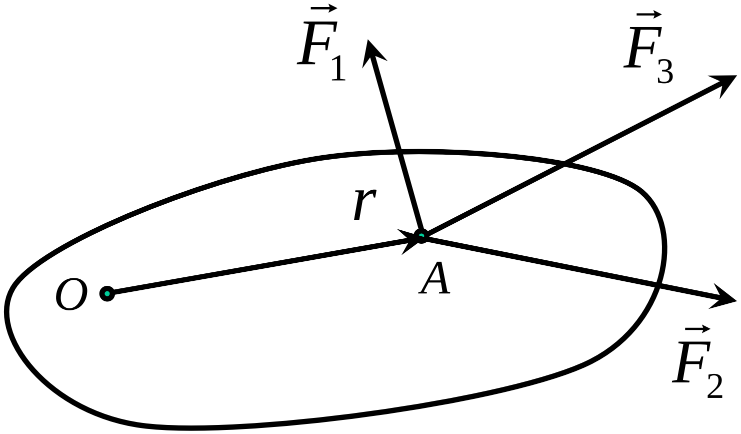
$$\vec{M}_\lambda = (\vec{M}_O \cdot \hat{n})\hat{n} = (\vec{r} \times \vec{F} \cdot \hat{n})\hat{n}$$

$$\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$$

$$(\vec{r} \times \vec{F} \cdot \hat{n}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} .(\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k})$$

$$= \begin{vmatrix} \alpha & \beta & \gamma \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} r_x & r_y & r_z \\ F_x & F_y & F_z \\ \alpha & \beta & \gamma \end{vmatrix}$$

Varignon's Theorem



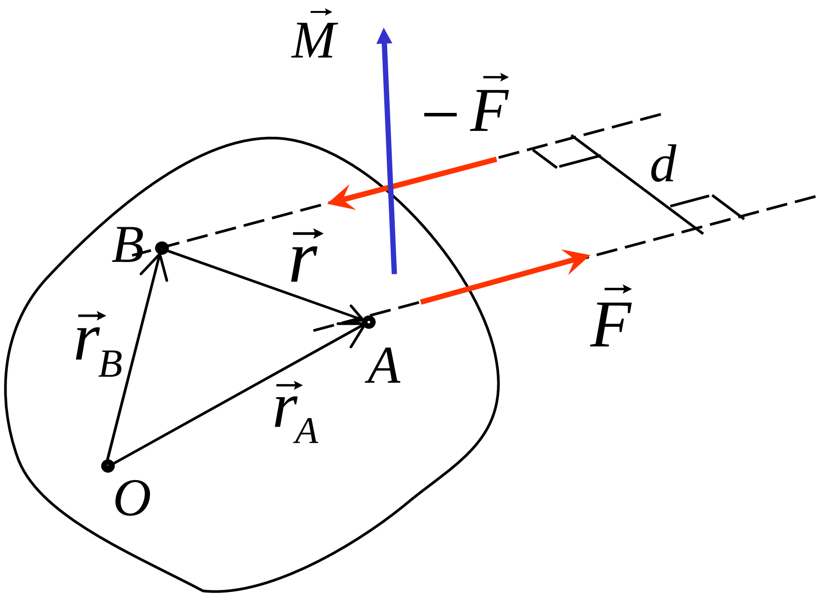
- Sum of the moments of a system of concurrent forces about a given point equals the moment of their sum about the same point

$$\vec{M}_o = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2 + \vec{r} \times \vec{F}_3 + \ldots = \vec{r} \times (\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots)$$

$$= \vec{r} \times (\sum \vec{F})$$

$$\boxed{\vec{M}_o = \sum(\vec{r} \times \vec{F}) = \vec{r} \times \vec{R}}$$

Couples(1)



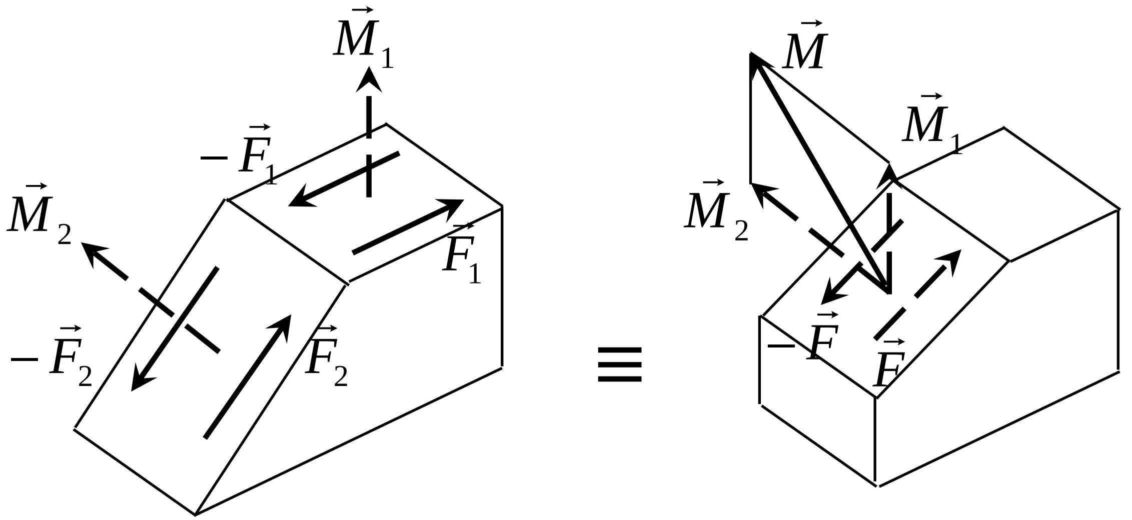
-Couple is a moment produced by two force of equal magnitude but opposite in direction.

$$\vec{M} = \vec{r}_A \times \vec{F} + \vec{r}_B \times (-\vec{F}) = (\vec{r}_A - \vec{r}_B) \times \vec{F}$$

$$\boxed{\vec{M} = \vec{r} \times \vec{F}}$$

- $\vec{r}$ = vector from any point on the line of action of $-\vec{F}$ to any point on the line of action of $\vec{F}$
- Moment of a couple is the same about all point → Couple may be represented as a free vector.
- Direction: normal to the plane of the two forces (right hand rule)
- Recall: Moment of force about a point is a sliding vector.

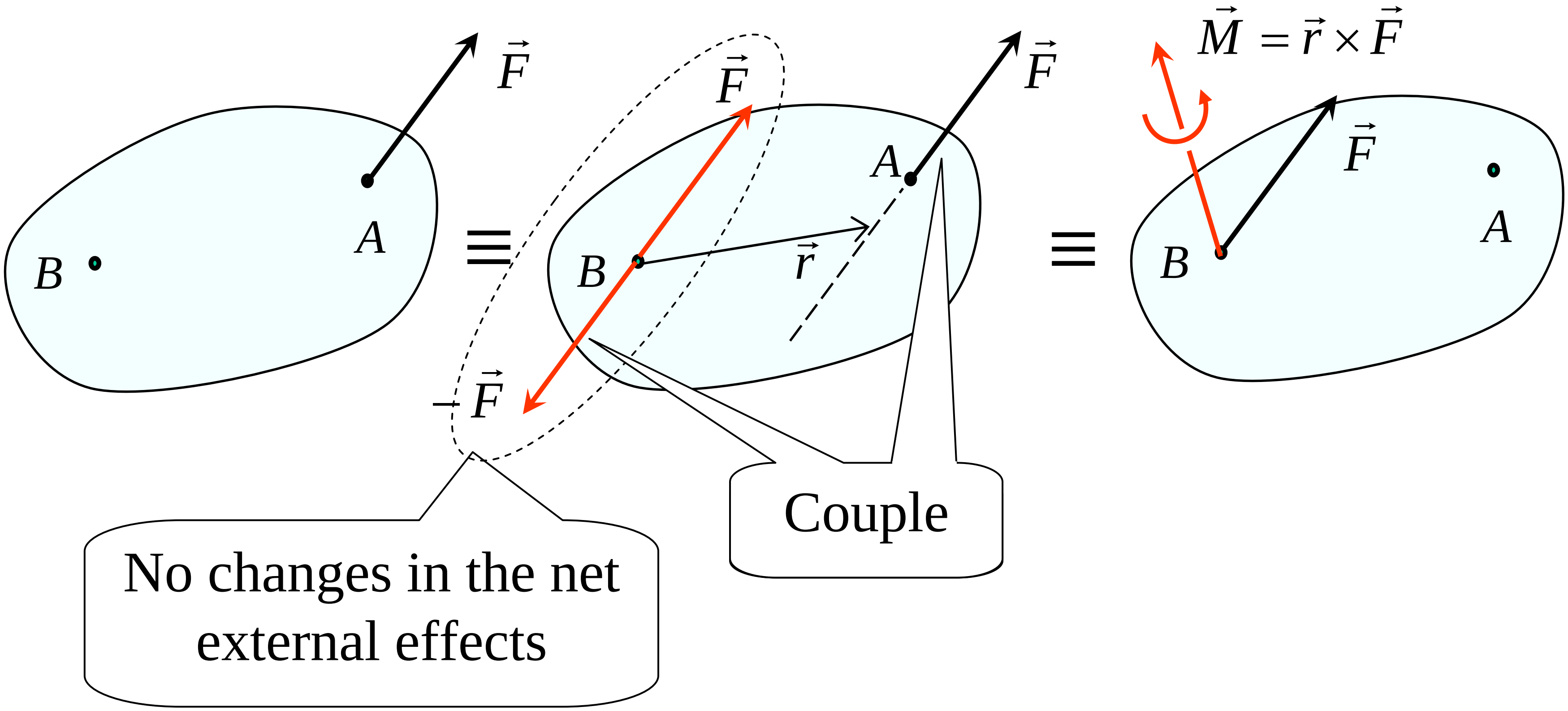
Couples(2)



[Couple from F_1]+[Couple from F_2] = [Couple from F_1+F_2]

couples are free vector. the line of action
or point of action are not needed!!!

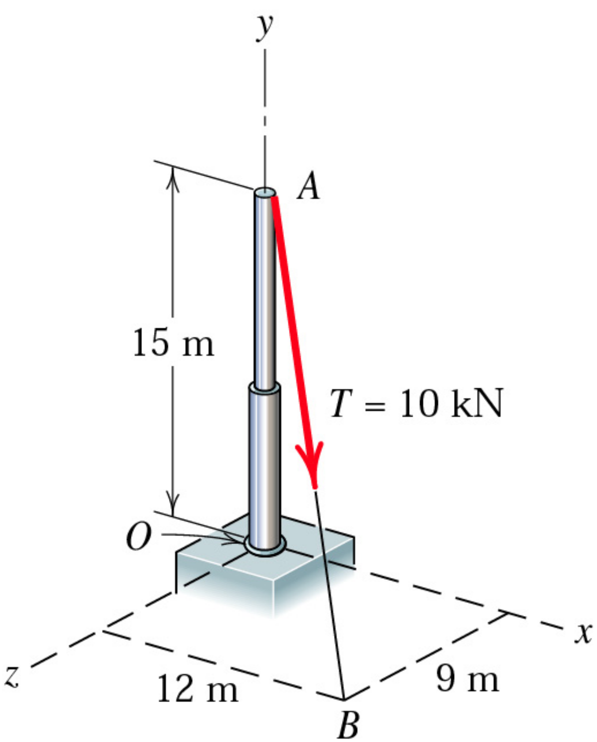
Force – couple systems



- $\vec{M} = \vec{r} \times \vec{F}$ = Moment of $\vec{F}$ about point *B*
- $\vec{r}$ is a vector start from point *B* to any point on the line of action of $\vec{F}$

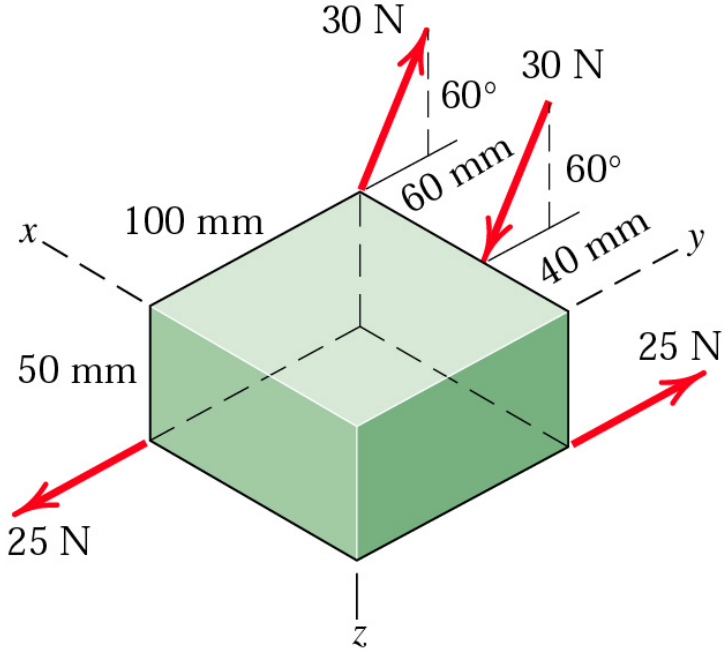
Sample 1

A Tension **T** of magnitude 10 kN is applied to the cable attached to the top *A* of the rigid mast and secured to the ground at *B*. Determine the moment M_z of T about the *z*-axis passing through the base *O*.



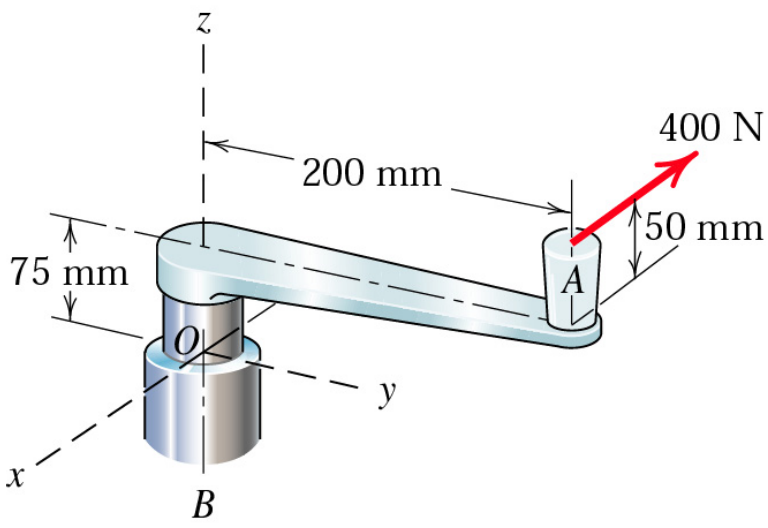
Sample 2

Determine the magnitude and direction of the couple **M** which will replace the two given couples and still produce the same external effect on the block. Specify the two force **F** and **–F**, applied in the two faces of the block parallel to the *y*-*z* plane, which may replace the four given forces. The 30-N forces act parallel to the *y*-*z* plane.



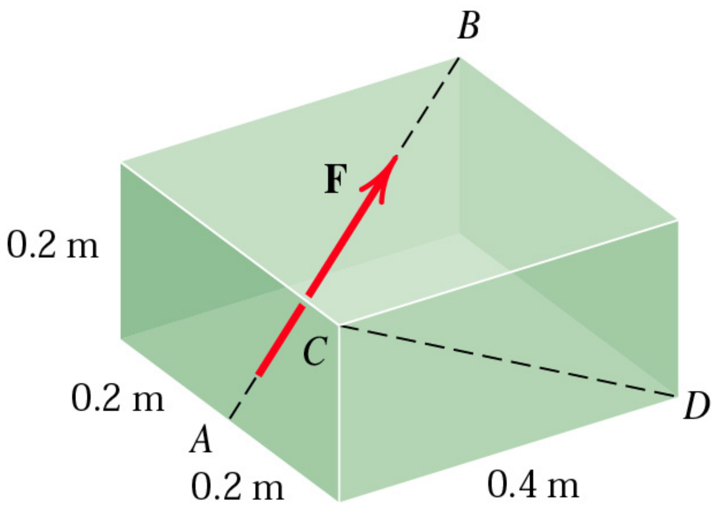
Sample 3

A force of 400 N is applied at *A* to the handle of the control lever which is attached to the fixed shaft *OB*. In determining the effect of the force on the shaft at a cross section such as that at *O*, we may replace the force by an equivalent force at *O* and a couple. Describe this couple as a vector **M**.



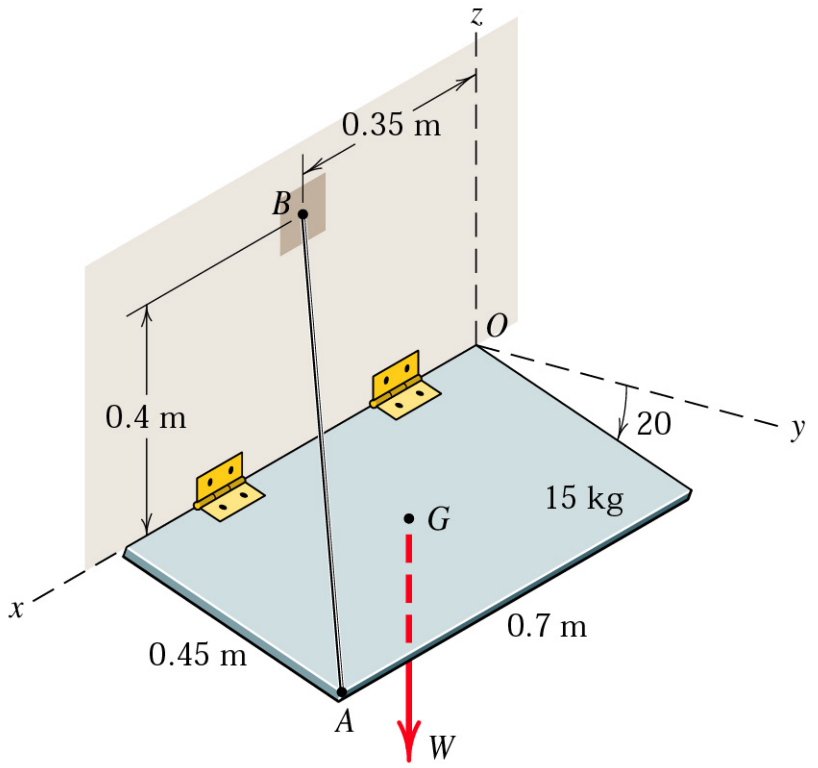
Sample 4

If the magnitude of the moment of **F** about line *CD* is 50 Nm, determine the magnitude of **F**.



Sample 5

Tension in cable AB is 143.4 N. Determine the moment about the x-axis of this tension force acting on point A . Compare your result to the moment of the weight W of the 15-kg uniform plate about the x-axis. What is the moment of the tension force acting at A about line OB



Summary (Force-Moment 3-D)

Force

1. Determine coordinate
2. Determine unit vector
3. Force can be calculate

Angle between force and x-,y-,z-axis

1. Force = $F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$
2. Determine amplitude of force F
3. $\cos\theta_x = F_x/F$, $\cos\theta_y = F_y/F$, $\cos\theta_z = F_z/F$

Angle between force and arbitrary axis

1. Determine unit vectors ($\mathbf{n}_F$, $\mathbf{n}$)
2. $\cos\theta = \mathbf{n}_F \cdot \mathbf{n}$

Summary (Force-Moment 3-D)

Moment ⇨ Consider to use vector method or scalar method

Vector method

Moment about an arbitrary point *O*

1. Determine **r** and **F**
2. Cross vector

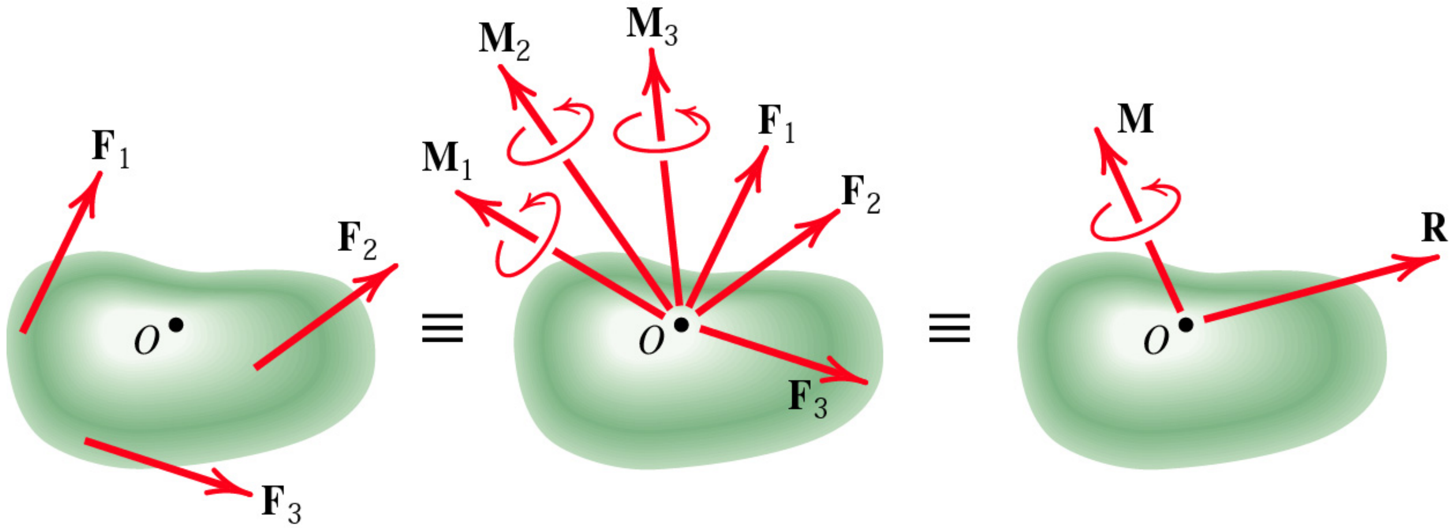
Moment about an arbitrary axis

1. Determine moment about any point on the axis $\mathbf{M}_O$
2. Determine unit vector of the axis **n**
3. Moment about the axis = $\mathbf{M}_O \cdot \mathbf{n}$

Angle between moment and axis

⤷ Same as angle between force and axis

Resultants(1)



Step1

Select a point to find moment

Step2

Replace forces with forces at point *O* + couples

Step3

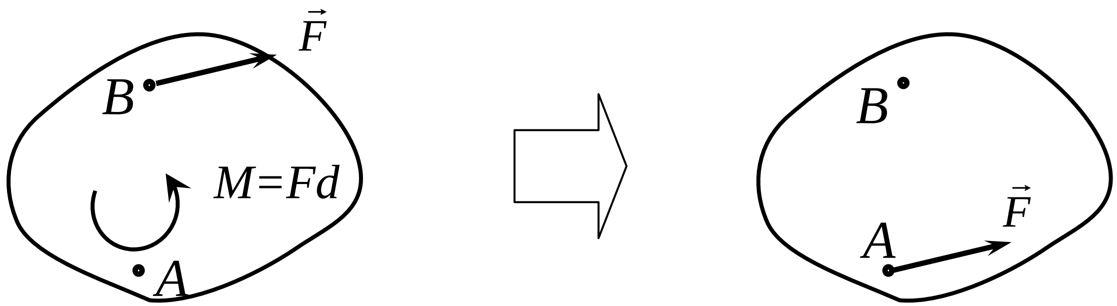
Add forces and couples vectorially to get the resultant force and moment

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots = \sum \vec{F}$$

$$\vec{M} = \vec{M}_1 + \vec{M}_2 + \vec{M}_3 + \ldots = \sum (\vec{r} \times \vec{F})$$

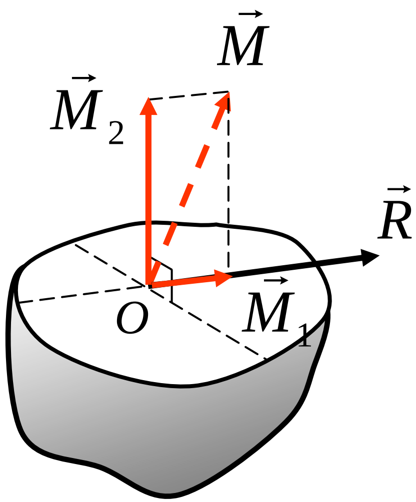
Resultants(2)

2-D



$\vec{M} \perp \vec{F}$ ⇒ Force + couple can be replaced by a force **F** by changing the position of **F**.

3-D



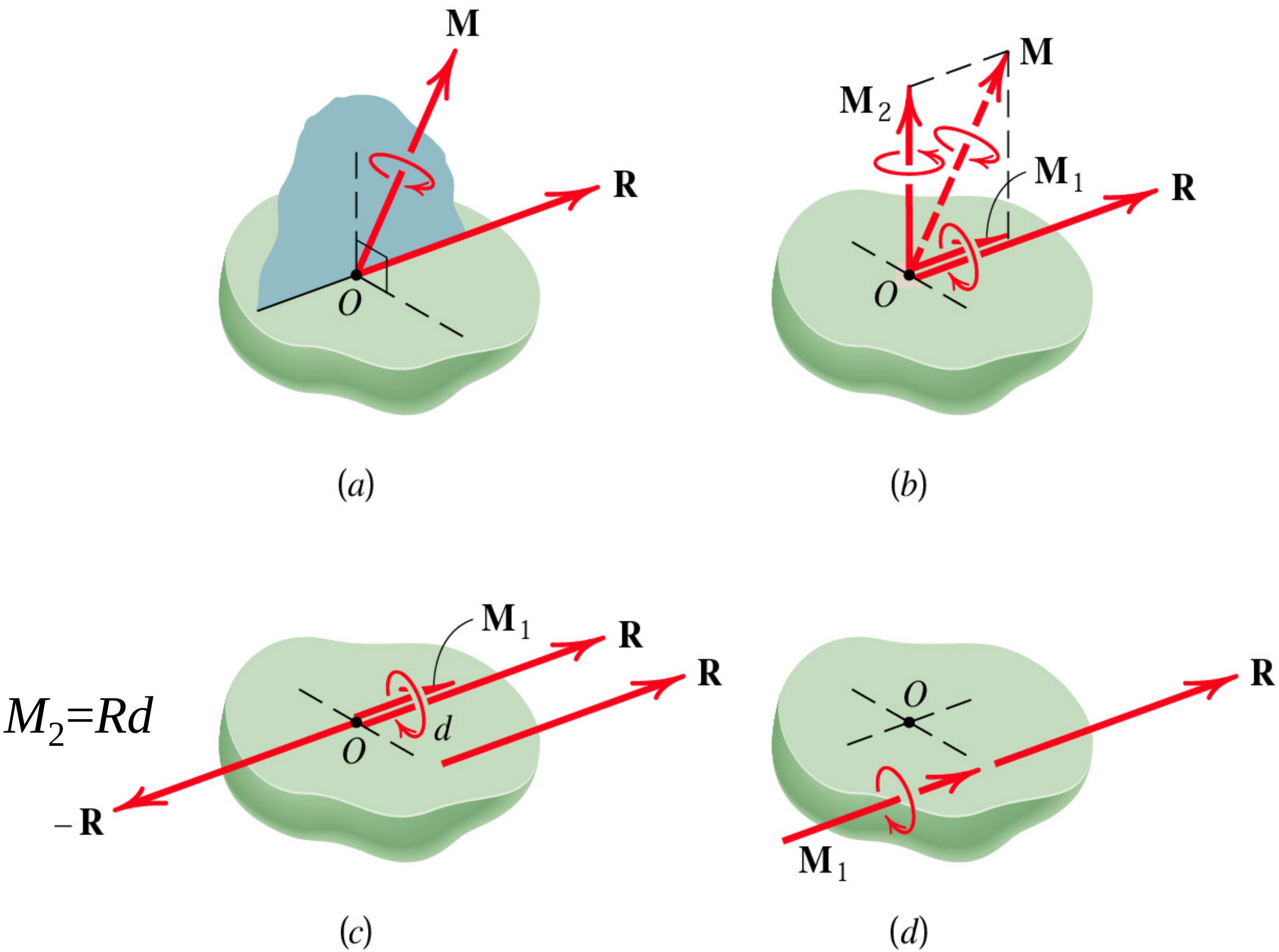
$\vec{M}_2 \perp \vec{R}$

$\mathbf{M_2}$ and **R** can be replaced by one force **R** by changing the position of **R**.

$\vec{M}_1 // \vec{R}$

$\mathbf{M_1}$ can not be replaced

Wrench resultant(1)

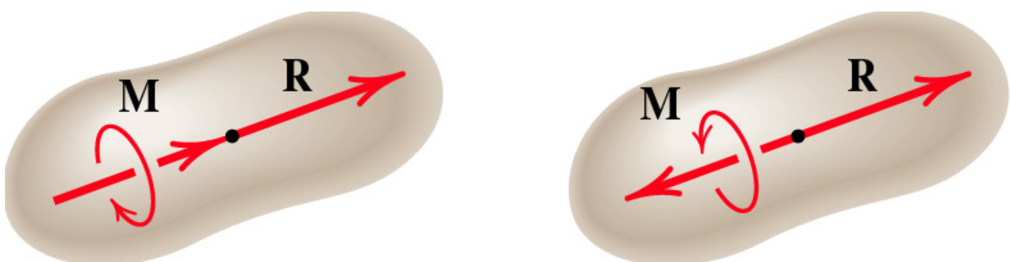


Wrench resultant(2)

2-D: All force systems can be represented with only one resultant force or couple

3-D: All force systems can be represented with a wrench resultant

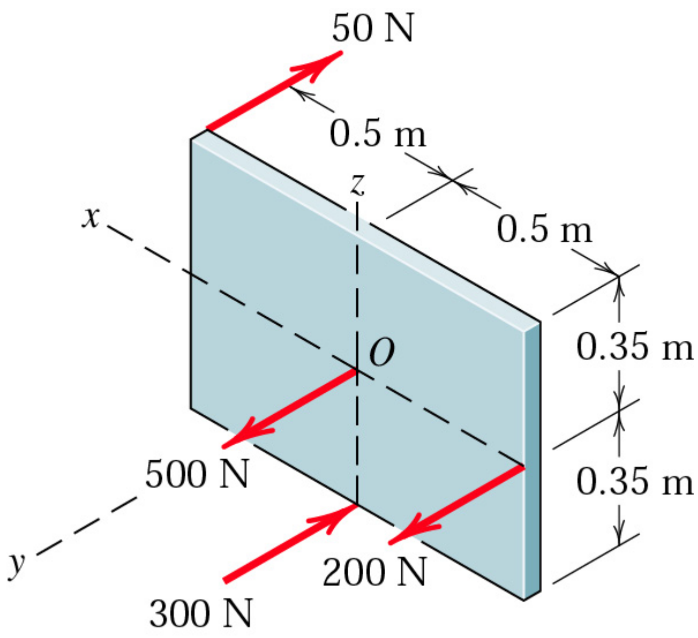
Wrench: resultant couple $\vec{M}$ parallel to the resultant force $\vec{R}$

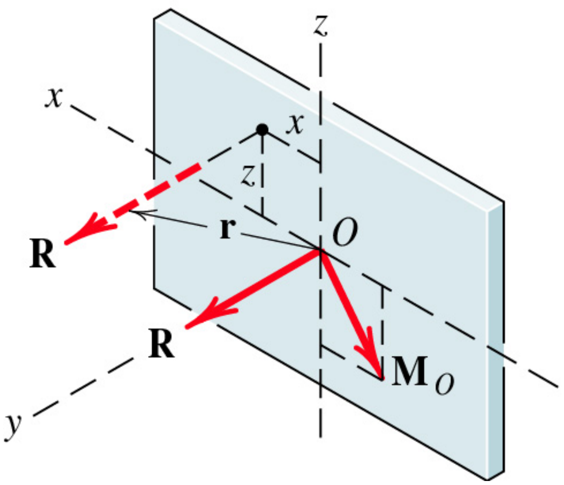


Positive wrench　　　Negative wrench

Sample 6

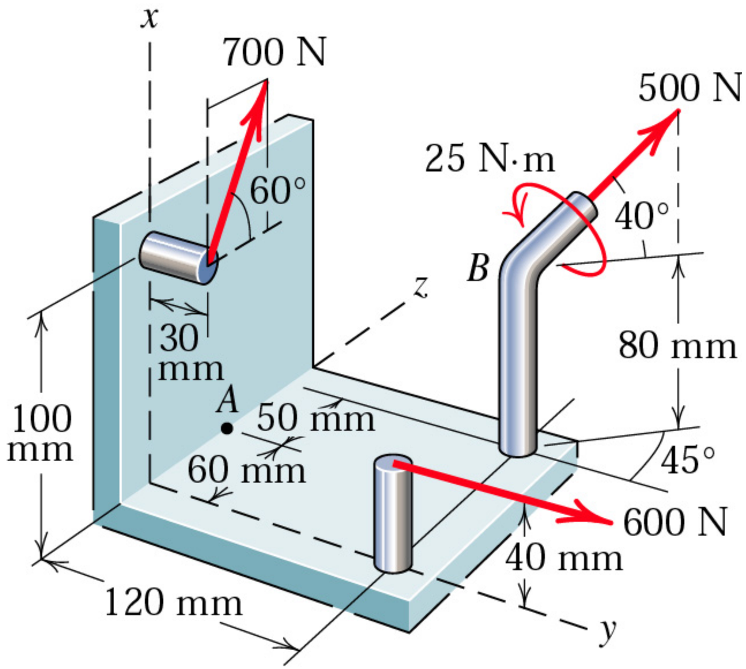
Determine the resultant of the system of parallel forces which act on the plate. Solve with a vector approach.

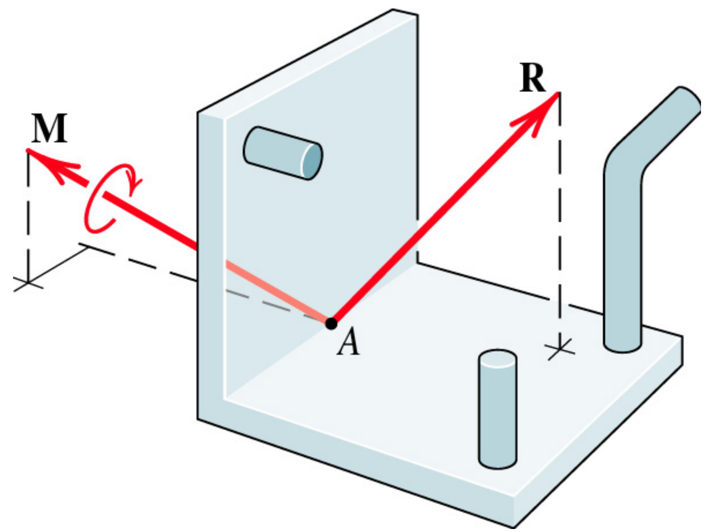




Sample 7

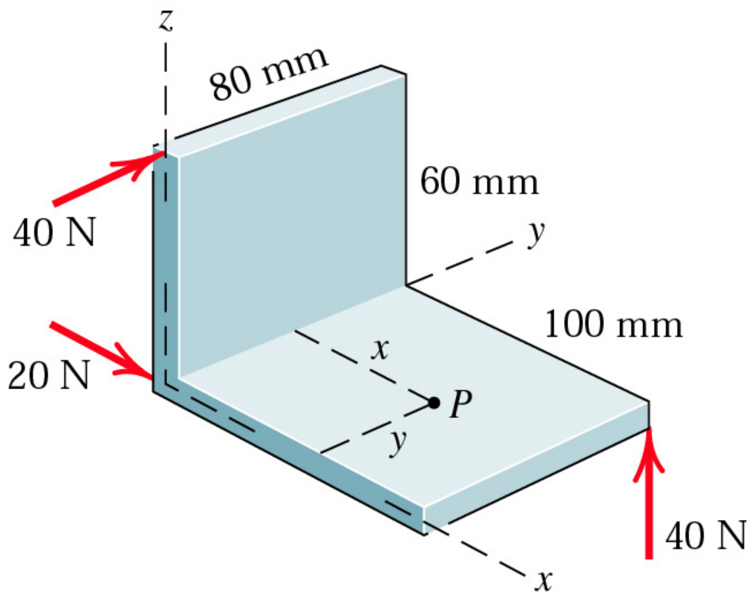
Replace the two forces and the negative wrench by a single force R applied at A and the corresponding couple M.

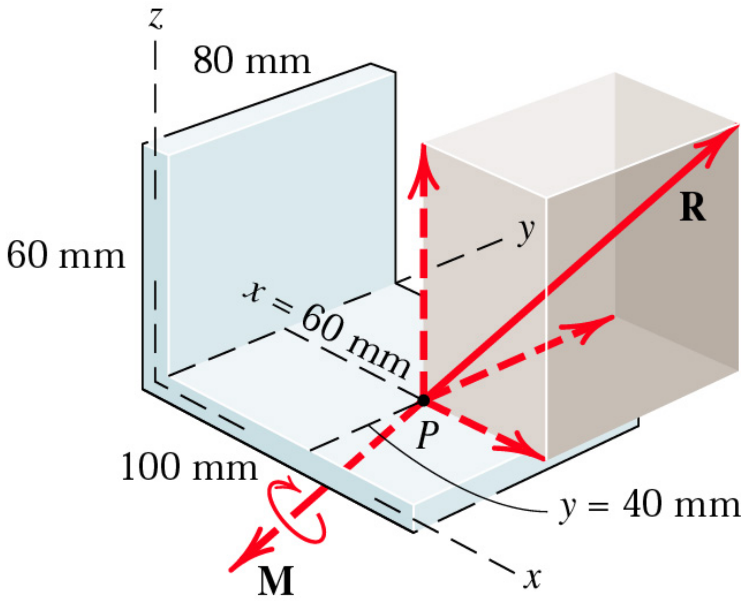




Sample 8

Determine the wrench resultant of the three forces acting on the bracket. Calculate the coordinates of the point *P* in the *x*-*y* plane through which the resultant force of the wrench acts. Also find the magnitude of the couple **M** of the wrench.





Sample 9

The resultant of the two forces and couple may be represented by a wrench. Determine the vector expression for the moment **M** of the wrench and find the coordinates of the point *P* in the *x*-*z* plane through which the resultant force of the wrench passes