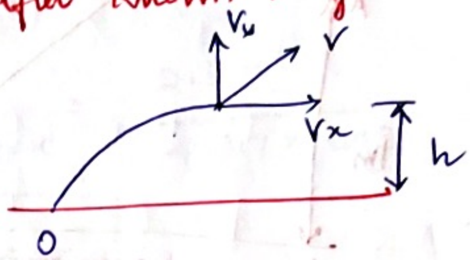
position of P

$x = u\cos\alpha t \qquad y = u\sin\alpha t - \frac{1}{2}gt^2$

Velocity and Direction of Projectile after known height



$V_y \Rightarrow$ To find.

$V_x = u\cos\alpha$

$V^2 = u^2 - 2gh$

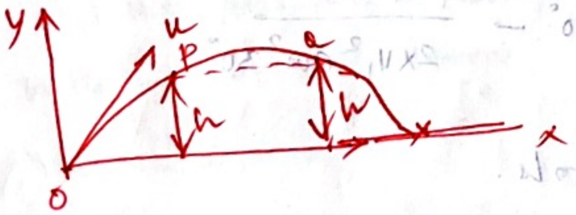
$V = V_y$

$V_y^2 = (u\sin\alpha)^2 - 2gh \qquad V = \sqrt{V_x^2 + V_y^2}$

$V_y = \sqrt{(u\sin\alpha)^2 - 2gh} \qquad \theta = \tan^{-1}\left(\frac{V_y}{V_x}\right)$

$\theta = \tan^{-1}\left[\frac{\sqrt{(u\sin\alpha)^2 - 2gh}}{u\cos\alpha}\right]$

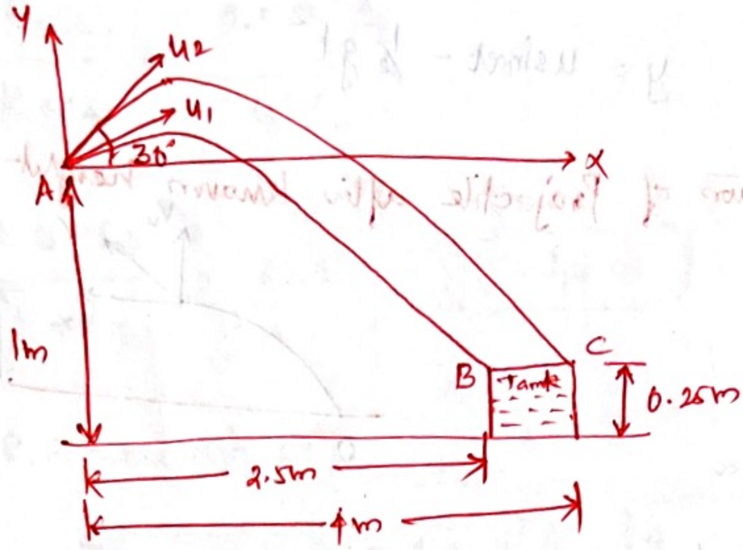
Time taken by projectile at a known height.



$h = ut - \frac{1}{2}gt^2$

$h = (u\sin\alpha)t - \frac{1}{2}gt^2$

(P) A boy throws two stones at an angle of 30° from point A as shown in diagram. Determine the time between throws so that both stones strikes the edges of the tank B and C at same instant. With what speed must he throw each stone?



soln :

stone (1) and (2) strikes at B and C.

Projectile (1)

Velocity $= u_1$ $\alpha = 30°$ Range $= 2.5$ m

Coordinates of B $(2.5, -0.75)$

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

$$-0.75 = 2.5\tan 30° - \frac{9.81\times 2.5^2}{2\times u_1^2\cos^2 30°}$$

$$u_1 = 4.317 \text{ m/s}.$$

Projectile (2)

Velocity $= u_2$ $\alpha = 30°$ $R = 4$ m

Coordinates of C $= (4, -0.75)$

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

$$-0.75 = 4\tan 30° - \frac{\cancel{g\times 4}\, 9.81\times 4^2}{2\times u_2^2\cos^2 90}$$

$$u_2 = 5.849 \text{ m/s}$$

e difference between throws.

Range = $u\cos\alpha \times t$

t_1 & t_2 ⇒ Time taken by stone

$R_1 = u_1 \cos\alpha\, t_1$ (or) $t_1 = \frac{R_1}{u_1\cos\alpha} = \frac{2.5}{4.317 \times \cos 30^\circ}$

$\boxed{t_1 = 0.668\,sec}$

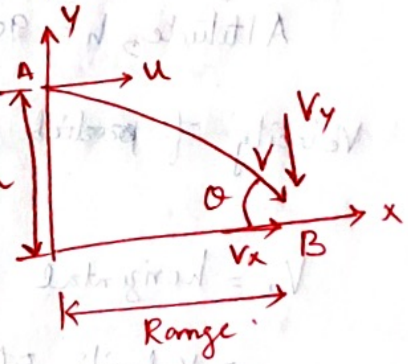
$R_2 = u_2 \cos\alpha\, t_2$ (or) $t_2 = \frac{R_2}{u_2\cos\alpha}$

$\boxed{t_2 = 0.789\,sec}$

Time difference = $t_2 - t_1$

$= 0.789 - 0.66 = 0.121\,sec$

Motion of Particle thrown horizontal from known height



At A : u ⇒ horizontal velocity with the particle is thrown. vertical velocity is zero

At B → V_y = vertical
V_x = horizontal

$\boxed{V_x = u}$

V_y ⇒ To find

$V = u + gt$

$V = V_y$ $\quad u = u\sin\alpha = 0$

$\boxed{V_y = gt}$

$V = \sqrt{V_x^2 + V_y^2}$

$= \sqrt{u^2 + (gt)^2}$

$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right)$

Range = horizontal Velocity x time taken

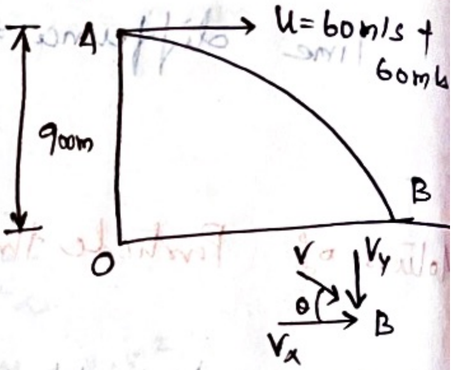
$R = u \times t$

$h = ut + \frac{1}{2}gt^2$

$u = u\sin\alpha = 0$

$h = 0 + \frac{1}{2}gt^2$ $\boxed{h = \frac{gt^2}{2}}$

(P) An aeroplane is flying horizontally with a consta speed of 60m/s at an altitude of 900m.. If the pilot d a package with the same horizontal speed of 60m/s.. Det the velocity when the package hits the ground and its ang with horizontal.



Velocity of aeroplane = 60m/s

$\left.\begin{array}{l}\text{horizontal velocity of}\\ \text{Package}\end{array}\right\} = 60m/s$

Altitude, $h = 900m$

$\left.\begin{array}{l}\text{Velocity of } \text{~~particle~~ package at}\\ B\end{array}\right\} V = \sqrt{V_x^2 + V_y^2}$

V_x = horizontal velocity at B

= Velocity of aeroplane + velocity of Package

= 60+60 = 120m/s.

V_y = Vertical velocity at B

To find V_y ⇒ time taken by the package to hit the ground.

$$h = ut + \frac{1}{2} g t^2$$

$$900 = 0 + \frac{1}{2} \times 9.81 \times t^2$$

$$\boxed{t = 13.54 \text{ sec}}$$

$$v = u + gt$$

$v = V_y \quad u = u\sin\alpha = 0$

$$V_y = 0 + (9.81 \times 13.54) = 132.83 \text{ m/s}$$

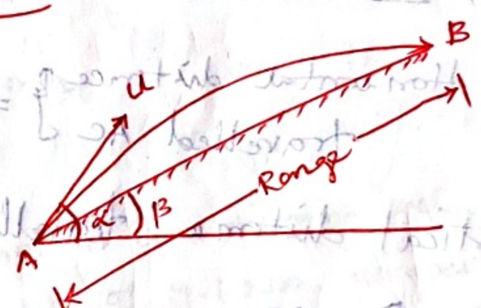
Velocity of Package at B $= \sqrt{V_x^2 + V_y^2}$

$$= \sqrt{120^2 + 132.83^2} = 179 \text{ m/s}.$$

Direction of velocity with horizontal $\Big\}\ \theta = \tan^{-1}\left(\frac{V_y}{V_x}\right)$

$$= \tan^{-1}\left(\frac{132.83}{120}\right) = 47.9^\circ.$$

Projectile up on Inclined plane :



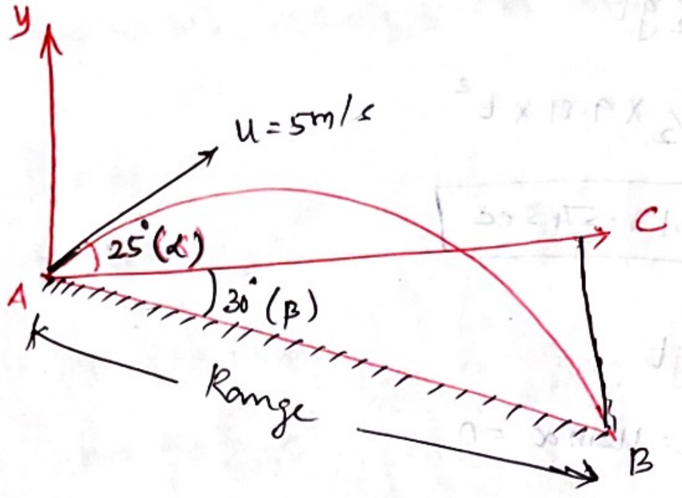
Time of flight

$$T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$$

Range of Projectile, $R = \dfrac{2u^2 \cos\alpha \sin(\alpha-\beta)}{g\cos^2\beta}.$

Max Range, $R_{max} = \dfrac{u^2}{g(1+\sin\beta)}.$

(P) A ball is projected from A with velocity 5 m/s at an angle 25° as shown. Determine the horizontal and vertical distances of B, which the ball hits the plane which is 30° below the horizontal.



<u>Given</u>

$u = 5\,m/s \quad \alpha = 25^\circ$ and $\beta = 30^\circ$

$$R = \frac{2u^2 \cos\alpha \sin(\alpha+\beta)}{g\cos^2\beta}$$

$$= \frac{2 \times 5^2 \times \cos 25 \sin(25+30)}{9.81 \times \cos^2 30} = 5.045\,m$$

Horizontal distance travelled

In $\triangle$ ABC

Horizontal distance travelled AC $\}$ $= AB \cos 30^\circ = 5.045 \times \cos 30^\circ$

$= 4.369\,m$

Vertical distance travelled

In $\triangle$ ABC

Vertical distance travelled BC $\}$ $= AB \sin 30^\circ = 5.045 \sin 30^\circ$

$= 2.522\,m.$

...inetics of Particles - Newton's laws of Motion.

...ewton's II law of Motion.

↳ Rate of change of Momentum is directly ...roportional to the impressed force and it takes place in ...e direction of force.

Change of Momentum = Final Momentum − Initial Momentum

$$= mv - mu$$
$$= m(v-u)$$

Momentum = Mass × Velocity = mv

$$\text{Rate of change of Momentum} = \frac{\text{Change of Momentum}}{\text{Time taken}}$$

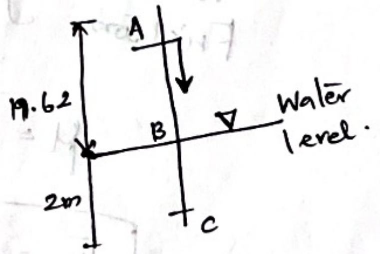
$$= \frac{m(v-u)}{t} \qquad \frac{v-u}{t} = a$$

$$= ma$$

external force ↘

$$P \propto ma$$

(P) A man of mass 60 kg jumps in a swimming pool, vertically downwards, from a height of 19.62m. He goes down in water by 2m, and then starts rising up. Calculate the average resistance of water.



$AB = 19.62m$ $BC = 2m$ $m = 60kg$

Motion from A to B

$u = 0$ $h = 19.62m$

v = Velocity of man, when he reaches the water level B.

$V^2 = u^2 + 2gh$

$V^2 = 0 + (2 \times 9.81 \times 19.62)$

$V = 19.62\, m/s.$

Motion from B to C.

$u = 19.62\, m/s \quad v = 0 \quad s = 2m$

$v^2 = u^2 + \cancel{2gh}\ 2as \qquad a \neq g$

$0 = 19.62^2 + 2 \times a \times 2$

$a = \dfrac{-(19.62)^2}{4} = 96.23\, m/s^2$

Average resistance, $P = ma$

$= 60 \times (-96.23)$

$= -5773\, N$

$\boxed{P = 5.77\, kN}$

U5

Frictional force :

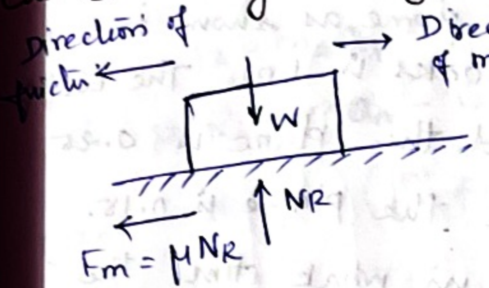
When 2 bodies are in contact with one another the property of two bodies by virtue of which a force is exerted between them at their point of contact to prevent one body from sliding on the other called "Frictional Force" or simply "Friction".

Coefficient of Friction $= \dfrac{\text{Limiting Friction}}{\text{Normal reaction}}$

$\mu = \dfrac{F_m}{N_R}$

$\boxed{F_m = \mu N_R}$

Case ① : Body moving on rough horizontal Range



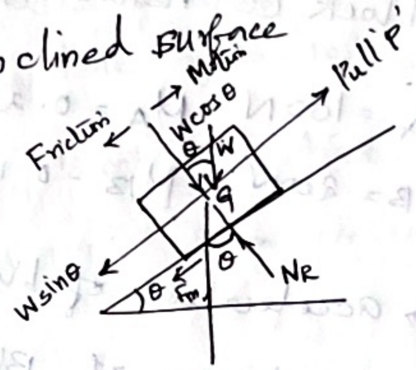
μ = Coefficient of friction

$F_m = \mu N_R$

$\boxed{F_\mu = \mu W}$

$\boxed{N_R = \mu}$

Case ② : Body pulled up on an inclined surface



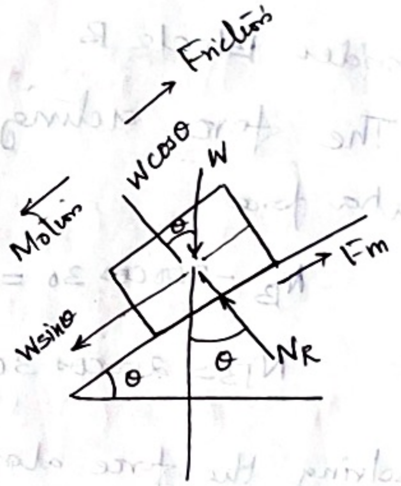
To find Fm

$$N_R - W\cos\theta = 0$$

$$\boxed{N_R = W\cos\theta}$$

$$F_m = \mu W\cos\theta$$

Case ③ : Body sliding downwards.



$$N_R - W\cos\theta = 0$$

$$\boxed{N_R = W\cos\theta}$$

$$F_m = \mu W\cos\theta$$

D'Alembert's Principle:

$$P = ma$$

P – External force

m – mass of moving body

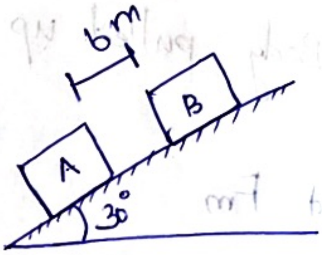
a = acceleration of body

It states that the system of forces acting on a body in motion is in dynamic equilibrium with inertia force of the body.

(P) Two blocks A and B of weight 100N and 200N resp. initially at rest on a 30° inclined plane as shown in diagram. The distance between the blocks is 6m. The coefficient of friction between the block A and the plane is 0.25 and that between the block B and the plane is 0.15. If they are released at same time, in what time the upper block (B) reaches the lower block (A).

$W_A = 100N \quad \mu_A = 0.25$

$W_B = 200N \quad \mu_B = 0.15$

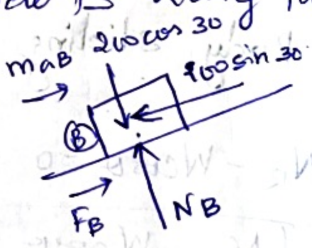


$a_A \Rightarrow$ acceleration of block A

a_B = acceleration of Block B.

Consider Block B

The force acting on the block B along the inertia force



$$N_B - 200\cos 30 = 0$$

$$N_B = 200\cos 30 = 173.2N$$

Resolving the force along the plane

$$F_B - 200\sin 30 + ma_B = 0$$

$$\mu_B N_B - 200\sin 30 + \left(\frac{200}{9.81} a_B\right) = 0$$

$$(0.15 \times 173.2) - 100 + 20.38\, a_B = 0$$

$$a_B = 3.63\ m/s^2$$

Consider Block A

Forces acting on the block A, along with the inertia force.