QPQA 14 marks questions

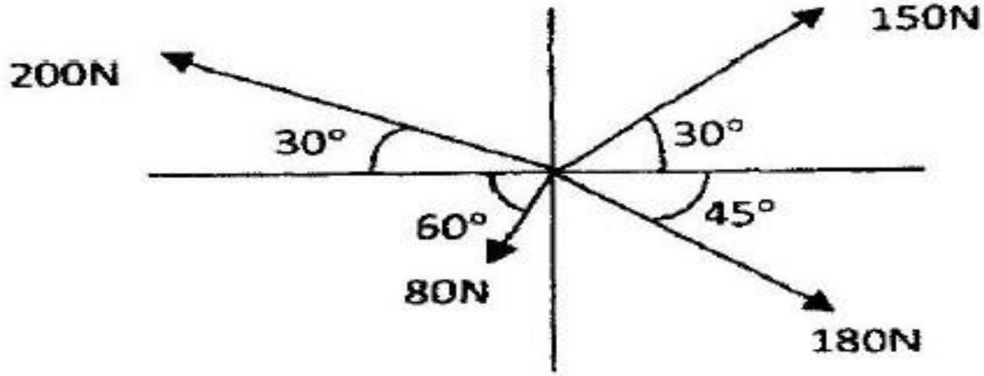
ENGINEERING MECHANICS

QP QA 14 marks questions

Solved Questions

QPQA 14 marks questions

Determine the resultant of the concurrent force system shown in the following Figure. **(AU JUN'10, DEC'10, DEC'12)**



Solution:

Resolving forces horizontally,

$\Sigma H = 150\cos 30 - 200\cos 30 - 80\cos 60 + 180\cos 45$

$= 130 - 173.2 - 40 + 127.28$

$\Sigma H = 44.08\ N$

Resolving forces vertically

$\Sigma V = 150\sin 30 + 200\sin 30 - 80\sin 60 - 180\sin 45$

$= 75 + 100 - 69.28 - 127.28$

$= \mathbf{-21.56\ N}$

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Magnitude of Resultant

$$R = \sqrt{\Sigma H^2 + \Sigma V^2}$$

$$= \sqrt{(44.08)^2 + (21.56)^2}$$

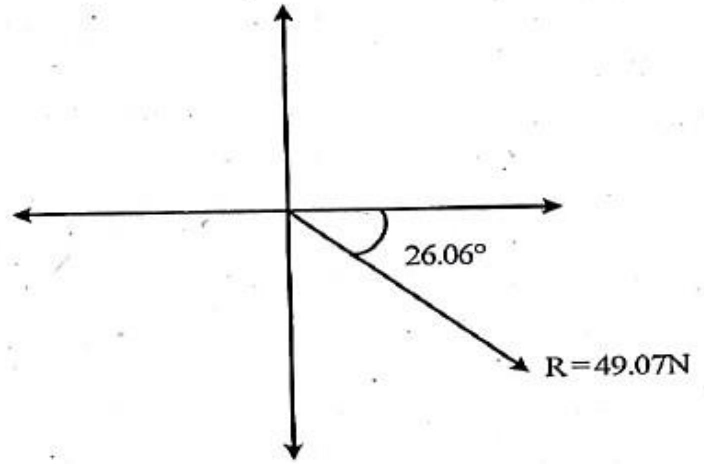
$$R = 49.07 \text{ N}$$

Direction of resultant is

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

$$\theta = \tan^{-1}\left(\frac{21.56}{44.08}\right) = 26.06 \text{ (IV Quadrant)}$$

The resultant force is shown in figure.



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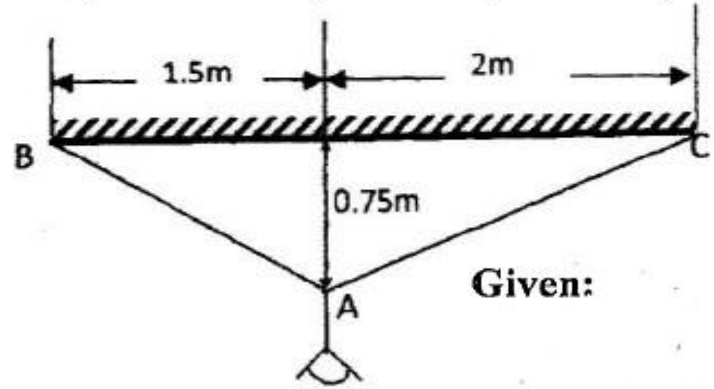
The following figure shows a 10 kg lamp supported by two cables AB and AC. Find the tension in each cable. **(AU JUN'10, DEC'10, DEC'12)**



Given:

mass of lamp = 10 kg

weight, w = 10×9.81 = 98.1 N

To find:

1) Tension in cable AB, T_{AB} = ?

2) Tension in cable AC, T_{AC} = ?

Solution:

From the given figure,

In angle ABD, $\tan\theta_1 = \frac{0.75}{1.5} = 26.56$

In angle ACD, $\tan\theta_2 = \frac{0.75}{2} = 20.55$

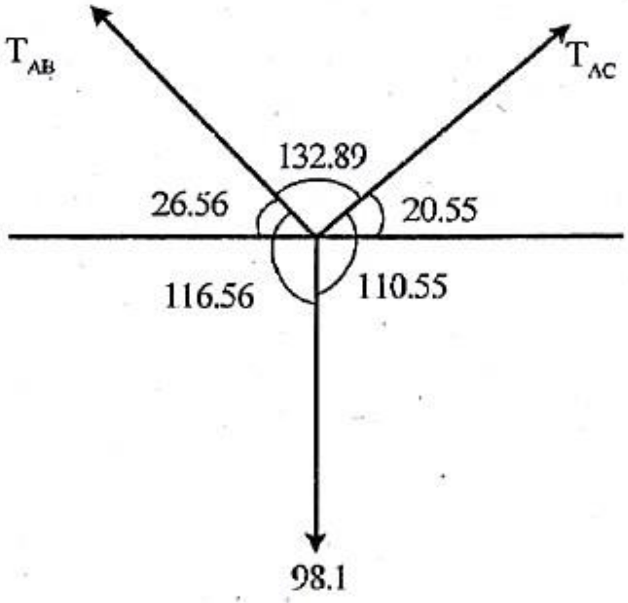
Solution:

From the given figure,

In angle ABD, $\tan\theta_1 = \dfrac{0.75}{1.5} = 26.56$

In angle ACD, $\tan\theta_2 = \dfrac{0.75}{2} = 20.55$

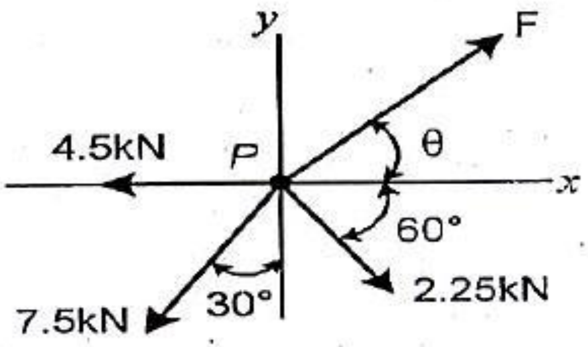
The system of forces is shown in figure.



Applying Lami's theorem

$$\frac{T_{AC}}{\sin 116.56} = \frac{T_{AB}}{\sin 110.55} = \frac{98.1}{\sin 132.89}$$

Determine the magnitude and angle θ and F so that particle shown in figure, is in Equilibrium **(AU MAY'11, JUN'12)**



Given :

Inclination of force, F = θ, with x-axis

Inclination of force 4.5KN = 0 with x-axis

Inclination of force 7.5KN with x-axis = (90–30) = 60°

Inclination of force 2.25KN with x-axis = 60°

To Find :

$F = ?\ ;\quad \theta = ?$

Solution :

It is given that the particle 'P' is in equilibrium. Hence

Hence, $\Sigma H = 0$ and $\Sigma V = 0$.

Resolving forces horizontally,

$$\Sigma H = F\cos\theta - 4.5\cos 0° - 7.5\cos 60° + 2.25\cos 60°$$

$$0 = F\cos\theta - 4.5 - 3.75 + 1.125$$

$$F\cos\theta = 7.125 \qquad \text{..... (1)}$$

Resolving forces vertically,

$$\Sigma V = F\sin\theta + 4.5\sin 0° - 7.5\sin 60° - 2.25\sin 60°$$

$$0 = F\sin\theta + 0 - 6.495 - 1.948$$

$$F\sin\theta = 8.443 \qquad \text{..... (2)}$$

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$$\frac{(2)}{(1)} \Rightarrow \frac{F\sin\theta}{F\cos\theta} = \frac{8.443}{7.125}$$

$$\tan\theta = 1.185$$

$$\theta = \mathbf{49.84^\circ}$$ **(Ans)**

Put $\theta = 49.84^\circ$ in eqn. (1), we get

$$F\cos(49.84) = 7.125$$

$$\mathbf{F = 11\ KN}$$ **(Ans)**

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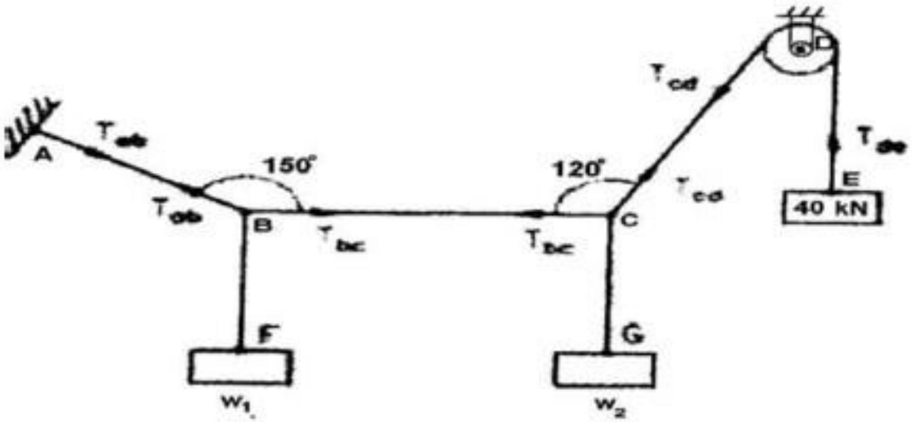
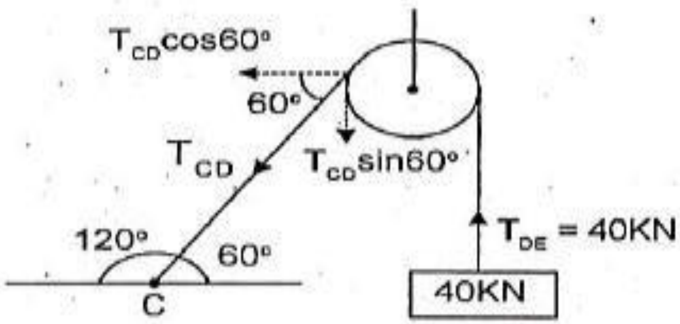
ABCDE is a light string whose end A is fixed. The weights W_1 and W_2 are attached to the string at B & C and the string passes round a small smooth wheel at D carrying a weight 40KN at the free end E. In the position of equilibrium, BC is horizontal and AB and CD make angles 150° and 120° with horizontal. **(AU DEC'12)**



Find (i) the tensions in AB, BC and DE of the given string (ii) magnitudes of W_1 and W_2.

Solution :

Let us consider the pulley first the various forces acting are shown below:



At point D,

$$T_{DE} = 40 \text{ kN}$$
$$\Sigma Fy = 0$$
$$-T_{CD} \sin60° \; T_{DE} = 0$$
$$-0.866 \; T_{CD} + 40 = 0$$
$$\mathbf{T_{CD} = 46.18 \text{ kN}} \qquad \textbf{(Ans)}$$

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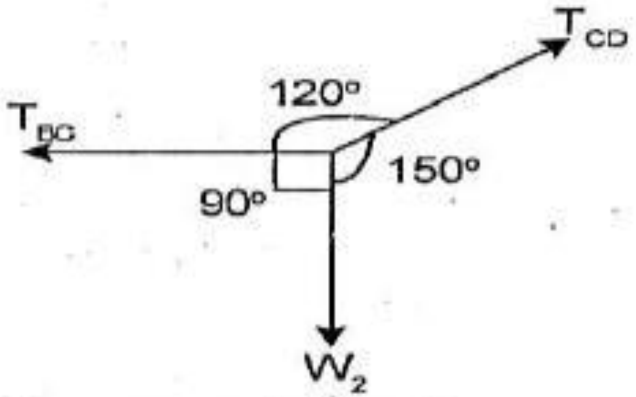
Now consider joint C, the various forces acting at C is shown below



Applying Lami's Theorem at joint C,

$$\frac{T_{CD}}{\sin 90°} = \frac{T_{BC}}{\sin 150°} = \frac{W_2}{\sin 120°}$$

$$\frac{46.18}{\sin 90°} = \frac{T_{BC}}{\sin 150°} = \frac{W_2}{\sin 120°}$$

$$T_{BC} = \frac{46.18 \times \sin 150}{\sin 90°} = 23 \text{ kN} \qquad \text{(Ans)}$$

$$W_2 = \frac{46.18 \times \sin 120}{\sin 90°} = 40 \text{ kN} \qquad \text{(Ans)}$$

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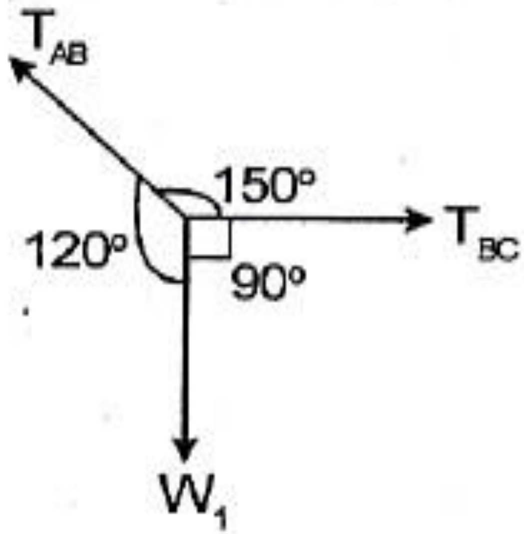
Now consider joint B, the various forces acting at 'B' is shown in fig. Applying Lami's Theorem,



$$\frac{T_{AB}}{\sin 90°} = \frac{W_2}{\sin 150°} = \frac{T_{BC}}{\sin 120°}$$

$$T_{AB} = \frac{T_{BC} \times \sin 90°}{\sin 120°} = 26.56 \text{ kN} \qquad \text{(Ans)}$$

$$W_1 = \frac{T_{BC} \times \sin 150}{\sin 120°} = 13.28 \text{ kN} \qquad \text{(Ans)}$$

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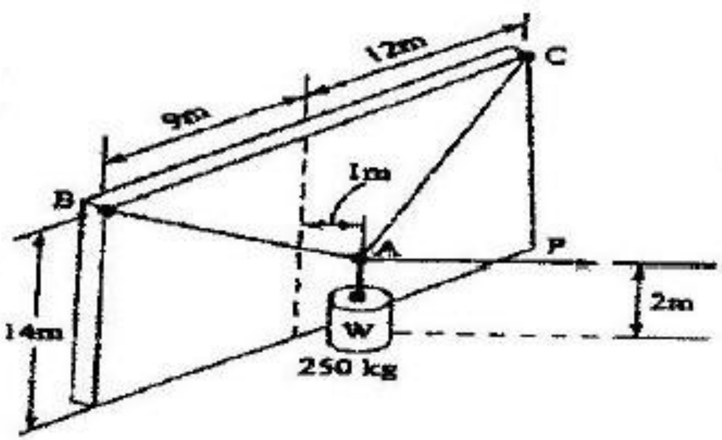
A horizontal force P normal to the wall holds the cylinder in the position shown in figure below. Determine the magnitude of P and the tension in each cable. **(AU DEC'12)**



Given:

Weight, W = 250kg = 250 x 9.81 = 2452.5N

To find:

i) Magnitude of P = ?

ii) Tension in each cable

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Solution:

Let T_{AB} and T_{AC} be the forces along the cables AB and AC respectively. The co-ordinates of various points are A(1, 2, 0), B(0, 14, 9), C(0, 14, –12).

Tension in Cable AB:

Position Vector $\overrightarrow{AB} = \vec{B} - \vec{A}$

$= (0i + 14j + 9k) - (i + 2j + 0k)$

$\overrightarrow{AB} = -i + 12j + 9k$

Magnitude of $\overrightarrow{AB} = \left|\overrightarrow{AB}\right|$

$= AB = \sqrt{(-1)^2 + 12^2 + 9^2}$

$= 15.03$

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Unit vector along AB,

$$n_{AB} = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$$

$$= \frac{-i + 12j + 9k}{15.03}$$

$$= -0.0665\ i + 0.7984\ j + 0.5988\ k$$

Tension along AB, $\overrightarrow{T_{AB}} = T_{AB} \cdot n_{AB}$

$$= T_{AB}\ (-0.665\ i + 0.7984j + 0.5988k)$$

$$= -0.665T_{AB}i + 0.7984T_{AB}j + 0.5988T_{AB}k$$

Tension in Cable AC:

Position Vector $\overrightarrow{AC} = \vec{C} - \vec{A}$

$$= (0i + 14j - 12k) - (i + 2j + 0k)$$

$$\overrightarrow{AC} = -i + 12j - 12k$$

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Magnitude of $\overrightarrow{AC} = |\overrightarrow{AC}|$

$$= AC = \sqrt{(-1)^2 + 12^2 + (-12)^2}$$

$$AC = 17$$

Unit vector along AC,

$$n_{AC} = \frac{\overrightarrow{AC}}{|\overrightarrow{AC}|}$$

$$= \frac{-i + 12j - 12k}{17}$$

$$= -0.0588\, i + 0.7058\, j - 0.7058k$$

Tension along AC, $\overrightarrow{T_{AC}} = T_{AC} \cdot n_{AC}$

$$= T_{AC}(-0.0588\, i + 0.7058\, j - 0.7058k)$$

$$= -0.0588T_{AC}i + 0.7058T_{AC}j - 0.7058T_{AC}k$$

Force through weight,

$$\overline{W} = W(-j) = -2452.5j$$

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Force through P,

$$\vec{P} = P \cdot i$$

using equations of equilibrium

$\Sigma F_x = 0$

$$-0.665\ T_{AB} - 0.0588\ T_{AC} + P = 0 \quad \text{.........} \ (1)$$

$\Sigma F_y = 0$

$$0.7984\ T_{AB} + 0.7058\ T_{AC} - 2452.5 = 0 \quad \text{.........} \ (2)$$

$\Sigma F_z = 0$

$$0.5988\ T_{AB} - 0.7058\ T_{AC} = 0 \quad \text{.........} \ (3)$$

Solving equations (1), (2), & (3) we get

$$T_{AB} = 1755.8\ N$$

$$T_{AC} = 1489.1\ N$$

$$P = 204.3\ N \quad \text{(Ans)}$$

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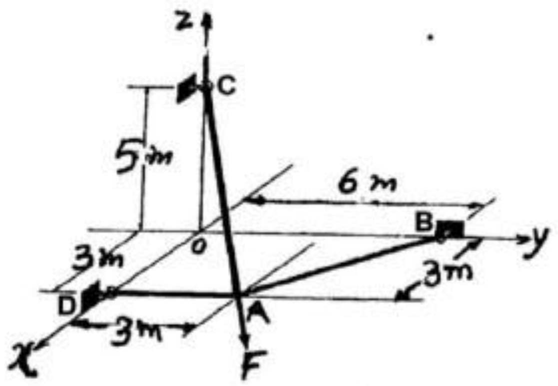
Figure below shows three cables AB, AC, AD that are used to support the end of a sign which exerts a force of $\vec{F} = \{250i + 450j - 450k\}N$ at A. Determine the force develop in each cable. **(AU DEC'11)**



Solution :

Let F_{AB}, F_{AC} and F_{AD} be the forces acting along cables AB, AC and AD respectively.

From the geometry of figure, the co-ordinates of various points are

A (3,0,3), B (6,0,0), C (0,5,0), D (0,0,3)

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Force Acting along AB

Position vector of AB is, $\overrightarrow{AB} = \overrightarrow{B} - \overrightarrow{A}$

$$= (6i - 0i + 0k) - (3i + 0i + 3k)$$

$$\overrightarrow{AB} = 3i - 3k$$

Magnitude of $\overrightarrow{AB} = |\overrightarrow{AB}| = AB = \sqrt{3^2 + (-3)^2} = 4.24$

Unit vector along AB is $n_{AB} = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$

$$= \frac{3i - 3k}{4.24}$$

$$n_{AB} = (0.7075i - 0.7075k)$$

Force in wire AB, $\overrightarrow{F_{AB}} = F_{AB} \cdot n_{AB}$

$$= F_{AB}\,(0.7075i - 0.7075k)$$

$$= 0.7075\ F_{AB}\ i - 0.07075\ F_{AB} k$$

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Force acting along AC

Position vector of AC is $\overrightarrow{AC} = \vec{C} - \vec{A}$

$$= (0i+5j+0k)-(3i+0j+3k)$$

$$\overrightarrow{AC} = -3i+5j-3k$$

Magnitude of $\overrightarrow{AC} = |\overrightarrow{AC}| = AC = \sqrt{(-3)^2+5^2+(-3)^2} = 6.55$

Unit vector acting along AC is $n_{AC} = \frac{\overrightarrow{AC}}{|\overrightarrow{AC}|}$

$$= \frac{3i+5j-3k}{6.55}$$

$$n_{AC} = -0.458i+0.763j\ -0.458k.$$

Force in wire AC, $\overrightarrow{F_{AC}} = F_{AC}.n_{AC}$

$$= F_{AC}(-0.458i+0.763j-0.458k)$$

$$\overrightarrow{F_{AC}} = -0.458\ F_{AC}i+00763\ F_{AC}j-0.458\ F_{AC}\ k$$

Force acting along AD

Position vector of AD is $\overrightarrow{AD} = \overrightarrow{D} - \overrightarrow{A}$

$$= (0i+0j+3k)-()3i+0j+3k)$$

$$\overrightarrow{AD} = -3i$$

Magnitude of $\overrightarrow{AD} = |\overrightarrow{AD}| = AD = \sqrt{(-3)^2} = 3$

Unit vector acting along AD is $n_{AD} = \frac{\overrightarrow{AD}}{|\overrightarrow{AD}|} = \frac{-3^{\circ}}{3} = -1^{\circ}$

Force in wire AD, $\overrightarrow{F_{AD}} = F_{AD}.n_{AD}$

$$\overrightarrow{F_{AC}} = -F_{AD}i$$

It is given that force at A is

$$\vec{F} = 250i + 450j - 150k$$

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It is given that force at A is

$$\vec{F} = 250i + 450j - 150k$$

Applying equations of equilibrium,

$$\Sigma Fx = 0.7075\ F_{AB} - 0.458\ F_{AC} - F_{AD} + 250 = 0 \quad ..... (1)$$

$$\Sigma Fy = 0.763\ F_{AC} + 450 = 0 \quad ..... (2)$$

$$\Sigma Fz = -0.7075\ F_{AB} - 0.458\ F_{AC} - 150 = 0 \quad ..... (3)$$

Solving equations (1), (2) & (3) we get,

$F_{AC} = 589.77N$ (Ans)

$F_{AB} = 593.8N$ (Ans)

$F_{AD} = 400N$ (Ans)

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A force (10i+20j-5k)N acts at a point P (4,3,2) m. Determine the moment of this force about the point Q(2,3,4) m in the vector form, Also find the magnitude of the moment andits angles with respect to x,y,z axes.**(AU Dec'10,JUN'12)**

Given :

Force, $\vec{F} = (10i + 20j - 5k)N$

P (4,3,2) m

Q (2,3,4) m

To Find :

Moment, $\overrightarrow{M_Q} = ?$

Magnitude of moment, $|\overrightarrow{M_Q}| = M = ?$

$\theta_x = ?,\ \theta_y = ?\ \theta_z = ?$

Solution :

Position vector, $\overrightarrow{QP} = \vec{P} - \vec{Q}$

$= (4i + 3j + 2k) - (2i + 3j + 4k)$

$\overrightarrow{QP} = 2i + 0j - 2k$

Moment about point Q, $\overrightarrow{M_Q} = \overrightarrow{QP} \times \vec{F} = \begin{vmatrix} i & j & k \\ 2 & 0 & 2 \\ 10 & 20 & -5 \end{vmatrix}$

$= i\ (0 + 40) - j\ (-10 + 20) + k\ (40 - 0)$

$\overrightarrow{M_Q} = (40i - 10j + 40k)$ **N-m.** **(Ans)**

Here, $M_x = 40$N-m ; $M_y = -10$ N-m, $M_z = 40$ N-m

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Magnitude of moment $= |\overline{M_O}| = M = \sqrt{(40)^2 + (-10)^2 + (40)^2}$

$$M = 57.44 \text{ N-m}$$

Angle made by $\overline{MO}$ with x-axis

$$\cos\theta x = \frac{Mx}{M}$$

$$\theta x = \cos^{-1}\left(\frac{40}{57.44}\right)$$

$$\theta x = 45.86^\circ$$ (Ans)

Similarly,

$$\cos\theta y = \frac{My}{M}$$

$$\theta y = \cos^{-1}\left(\frac{-10}{57.44}\right)$$

$$\theta y = 100.02^\circ$$

(Ans)

$$\theta z = \cos^{-1}\left(\frac{Mz}{M}\right) = \cos^{-1}\left(\frac{40}{57.44}\right)$$

$$\theta z = 45.86^\circ$$ (Ans)

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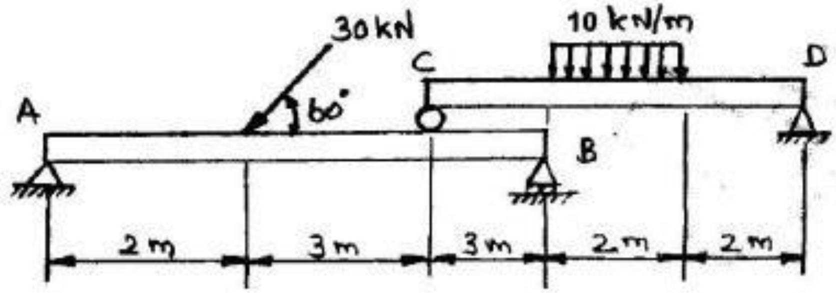
Two beams AB and CD are shown in figure. A and D are hinged supports. B and C are roller supports.



(i) Sketch the free body diagram of the beam AB and determine the reactions at the supports A & B.

(ii) Sketch the free body diagram of beam AB and determine the reactions at the supports C and D. **(AU Dec'10,DEC'12)**

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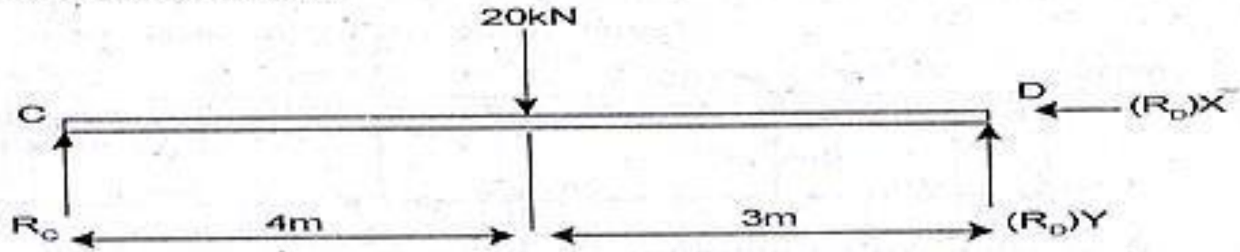
Solution :

Let R_A, R_B, R_C and R_D be the reactions at supports A, B, C and D respectively.

Consider the beam CD. The free body diagram of beam CD is shown below.



The uniformly distributed load of 10 kN/m for a length of 2 m is assumed as equivalent point load of (10×2=20kN) and acting at a distance 4 m form C.

Using equations of equilibrium,

$\Sigma F_x = 0$

$$(R_D)_x = 0 \qquad \textbf{(Ans)}$$

$\Sigma F_y = 0$

$$R_C - 20 + (R_D)_y = 0$$

$$R_C + (R_D)_y = 20 \qquad \ldots\ (1)$$

Taking moments about C,

$\Sigma M_C = 0$

$$-20\times4 + (R_D)_y \times 7 = 0$$

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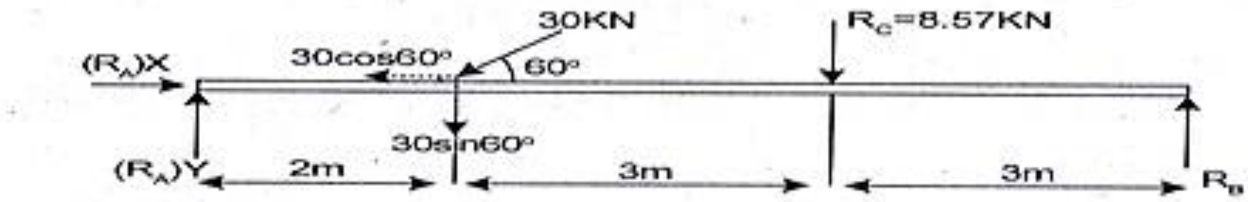
$$-80+7(R_D)_y = 0$$
$$(R_D)_y = 11.43 \text{ kN} \quad \text{(Ans)}$$
Put $(R_D)_y = 11.43$ in eqn. (1), we get
$$R_C + 11.43 = 20$$
$$R_C = 8.57 \text{ kN} \quad \text{(Ans)}$$

Now consider the beam AB. The free body diagram of beam AB is shown below



Using equations of equilibrium,
$$\Sigma F_x = 0$$
$$(R_A)_x - 30\cos 60 = 0$$
$$(R_A)_x = 15 \text{ kN}$$
$$\Sigma F_y = 0$$
$$(R_A)_y - 30\sin 60 - 8.57 + R_B = 0$$
$$(R_A)_y + R_B = 34.55 \quad \text{..... (2)}$$
Taking moments about A,
$$\Sigma M_A = 0$$
$$-30\sin 60\times 2 - 8.57\times 5 + R_B\times 8 = 0$$
$$-51.96 - 42.85 + 8R_B = 0$$
$$R_B = 11.85 \text{ kN} \quad \text{(Ans)}$$

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Put $R_B = 11.85$ in eqn. (2), we get

$$(R_A)_y + 11.85 = 34.55$$

$$(R_A)_y = 22.7 \text{ kN}$$

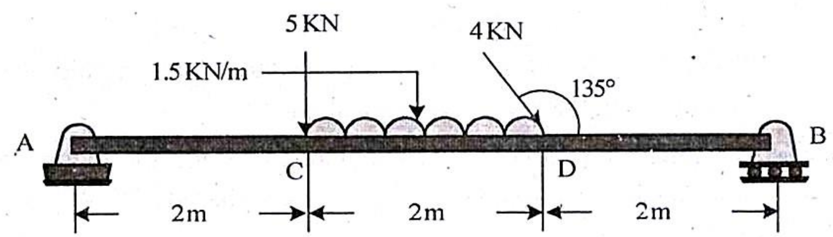
$$R_A = \sqrt{(R_A)_x{}^2 + (R_A)_y{}^2}$$

$$= \sqrt{(15)^2 + (22.7)^2}$$

$$R_A = 27.2 \text{ kN} \qquad \text{(Ans)}$$

QPQA 14 marks questions

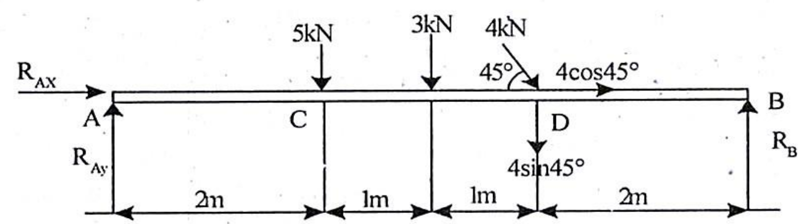
A simply supported beam AB of 6m span is loaded as shown A is a hinged support; B is a roller support. Determine the reactions at A and B. **(AU MAY'11)**



Solution :

Let R_{AX} and R_{AY} be horizontal and verticle component of reaction R_A at hinged support A.

Let R_B be the vertical component at B due to roller support.



The uniformly distributed load of 1.51kN/m for a length of 2m is assumed as equivalent point load of $(1.5\times2= 3kN)$ and acting at a distance $\frac{2}{2} = 1m$ from 'C'.

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Using equations of equilibrium

Taking moments about A,

$\Sigma M_A = 0$

$$-5\times2-3\times3-4\sin 45^\circ\times4 + R_B\times6 = 0$$

$$-10-9-11.31+6R_B = 0$$

$$\mathbf{R_B = 5.05kN} \quad \textbf{(Ans)}$$

$\Sigma F_x = 0$

$$R_{Ax}+4\cos 45^\circ = 0$$

$$R_{Ax} = -3.98 \text{ kN}$$

$\Sigma F_y = 0$

$$R_{Ay}-5-3-4\sin 45^\circ + 4R_B = 0$$

$$R_{Ay} = 5.778\text{kN}$$

Magnitude of Reaction, RA $= \sqrt{R_{AX}{}^2 + R_{Ay}2}$

$$= \sqrt{(3.98)^2 + (5.77)^2}$$

$$\mathbf{R_A = 7 \text{ kN}} \quad \textbf{(Ans)}$$

Direction of R_A is

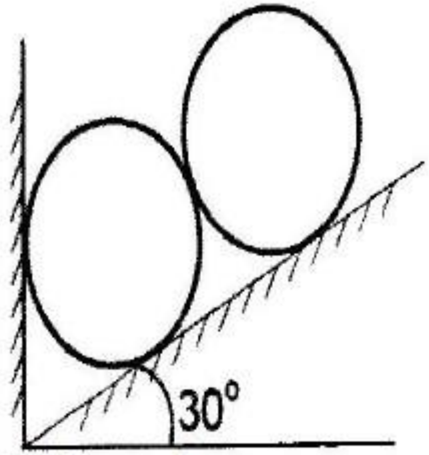
$$\theta = \tan^{-1}\left(\frac{R_{Ay}}{R_{Ax}}\right)$$

$$\theta = \tan^{-1}\left(\frac{5.778}{3.98}\right)$$

$$\theta = \mathbf{55.4} \quad \textbf{(Ans)}$$

QPQA 14 marks questions

! Two identical rollers, each of weight 500N, are supported by an inclined plane making an angle of 30° to the horizontal and a vertical wall as shown in the figure. **(AU Jun'10,DEC'12)**



(i) Sketch the free body diagrams of the two rollers.
(ii) Assuming smooth surfaces, find the reactions at the support points.

Given:

Weight of two identical rollers, W = 500 N

To Find:

Reactions at supports.

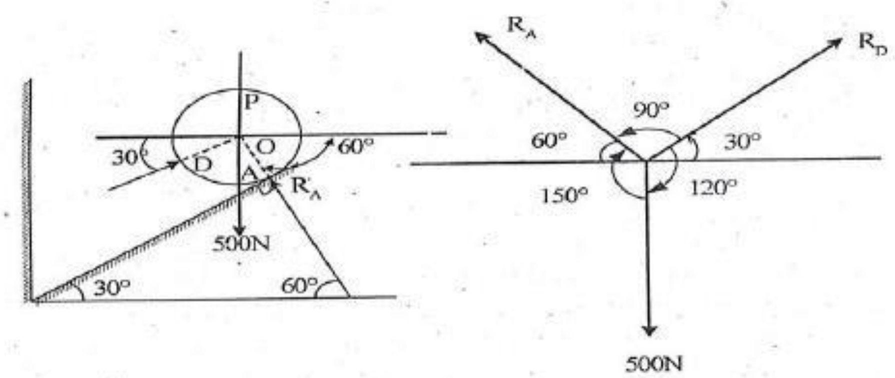
Solution:

Let R_A, R_B and R_C be the reactions at supports.

Considering the roller P

The freebody diagram is shown in the figure.

Free body diagram **Force diagram**



Applying Lami's theorem

$$\frac{R_A}{\sin 120} = \frac{R_B}{\sin 150} = \frac{500}{\sin 90}$$

$$\frac{R_A}{\sin 120} = \frac{500}{\sin 90} \Rightarrow \mathbf{R_A = 433\ N}$$

$$\frac{R_D}{\sin 150} = \frac{500}{\sin 90} \Rightarrow \mathbf{R_D = 250\ N}$$

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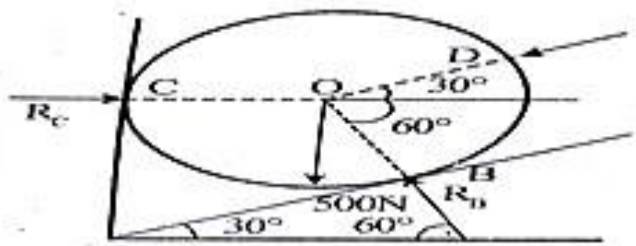
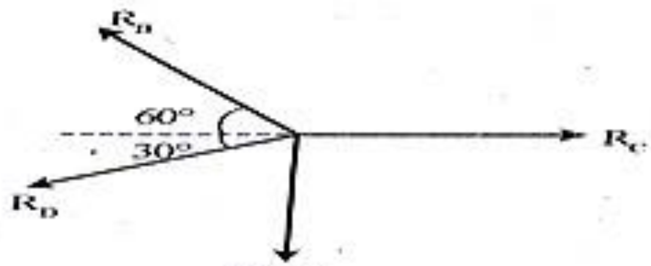
Considering rollers Q.

The free body diagram is shown in the figure.

Free body Diagram



Force Diagram



Applying equations of equilibrium,

$\Sigma F_x = 0$

$$R_C - R_B \cos 60 - R_D \cos 30 = 0$$
$$R_C - 0.5\ R_B - 250 \times 0.866 = 0$$
$$R_C - 0.5\ R_B = 216.5 \quad ....(1)$$

Resolving forces vertically,

$\Sigma F_y = 0$

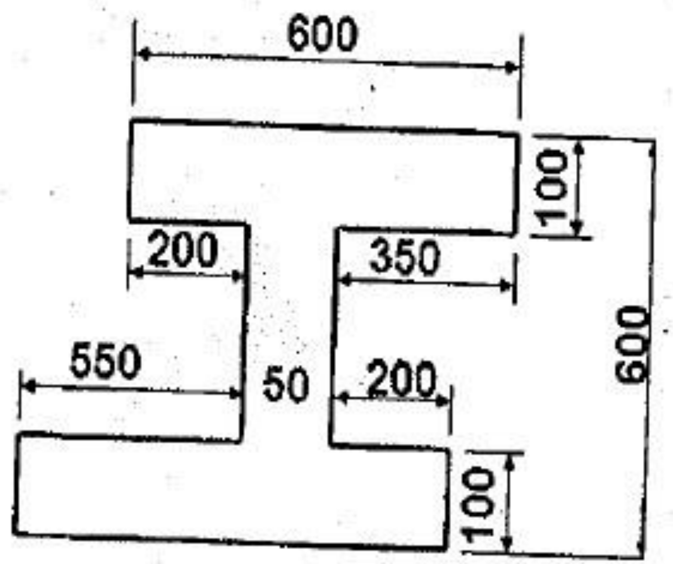
$$R_B \sin 60 - R_D \sin 30 - 500 = 0$$
$$0.866\ R_B - 250 \times 0.5 - 500 = 0$$
$$R_B = 721.7\ N$$

Put $R_B = 721.7$ in equation (1)

$$R_C - 0.5\ (721.7) = 216.5$$
$$R_C = 577.35\ N$$

QPQA 14 marks questions

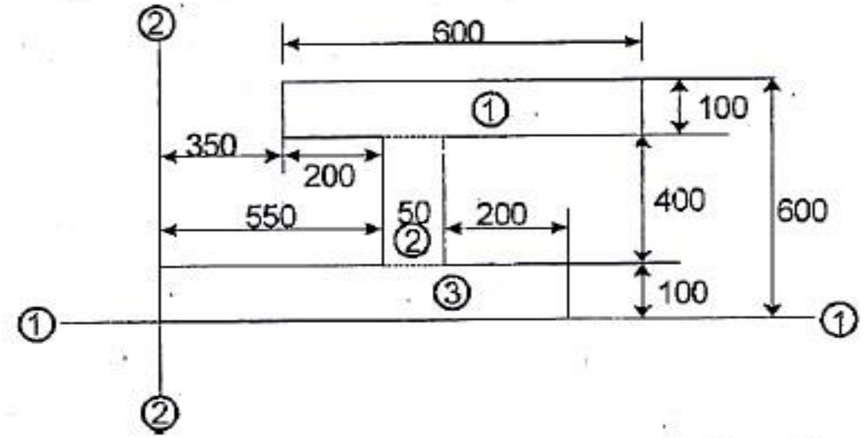
For the section shown in figure below, locate the horizontal and vertical centroidal Axis (**AU JUN'12)**



QPQA 14 marks questions

Solution :

The given section is split into three rectangles as shown in the figure. Let 1-1 and 2-2 be the reference axis



Component	Area a (mm^2)	Centroidal Distance from 2-2 axis 'x' (mm)	Centroidal Distance from 1-1 axis 'y' (mm)	ax (mm^2)	ay (mm^2)
Rectangle (1)	100×600 = 60000	$\frac{600}{2}+350 = 650$	$\frac{100}{2}+500 = 550$	39×10^6	33×10^6
Rectangle (2)	400×50 = 20000	$\frac{50}{2}+550 = 575$	$\frac{400}{2}+100 = 300$	11.5×10^6	6×10^6
Rectangle (3)	800×100 = 80000	$\frac{800}{2} = 400$	$\frac{100}{2} = 50$	32×10^6	4×10^6
	$\Sigma a = 160 \times 10^3$			$\Sigma ax = 82.5\times10^6$	$\Sigma ay = 43\times10^6$

QPQA 14 marks questions

Centroidal distance from reference axis, 2–2,

$$\bar{x} = \frac{\sum ax}{\sum a}$$

$$= \frac{82.5 \times 10^6}{160 \times 10^3}$$

$$\bar{x} = 515.625 \text{ mm} \qquad \text{(Ans)}$$

Centroidal distance from reference axis, 2–2,

$$\bar{y} = \frac{\sum ay}{\sum a}$$

$$= \frac{43 \times 10^6}{160 \times 10^3}$$

$$\bar{y} = 268.75 \text{ mm} \qquad \text{(Ans)}$$

QPQA 14 marks questions

Calculate the centroidal polar moment of inertia of a rectangular section with breadth of 100 mm and height 200 mm. **(AU DEC'10,JUN'12)**

Given :

For Rectangular section

Breadth, $b = 100mm$

Height, $h = 200mm$

To Find :

Polar moment of Inertia, $J = ?$

Solution :

M.I of rectangle about x-x axis

$$I_{xx} = \frac{bh^3}{12} = \frac{100 \times 200^3}{12} = 66.67 \times 10^6 mm^4$$

M.I of rectangle about y-y axis

$$I_{yy} = \frac{hb^3}{12} = \frac{200 \times 100^3}{12} = 16.67 \times 10^6 mm^4$$

Polar moment of Inertia is,

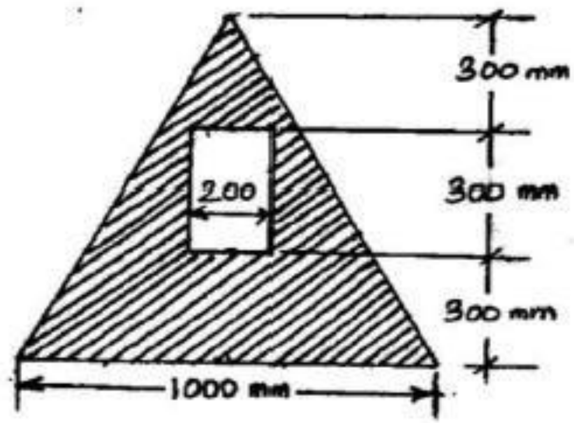
$$J = I_{xx} + I_{yy}$$

$$= 66.67 \times 10^6 + 16.67 \times 10^6$$

$$\mathbf{J = 83.34 \times 10^6 mm^4}$$ **(Ans)**

QPQA 14 marks questions

Find the moment of inertia of the shaded area shown in figure about the vertical and horizontal centroidal axes. The width of the hole is 200 mm. **(AU DEC'12, JUN'10)**



As the given section is symmetrical about y-y axis its centroid will lie on this axis. Let 1-1 be the reference axis

QPQA 14 marks questions

As the given section is symmetrical about y-y axis its centroid will lie on this axis. Let 1-1 be the reference axis

Component	Area a (cm^2)	Centroidal Distance from 1-1 axis 'y' (cm)	ay (cm^2)	I_{self} about the axis x-x (cm^4)
Triangle	$a_1 = \frac{1}{2} bh$ $= \frac{1}{2} \times 90 \times 100$ $= 4500$	$y_1 = \frac{h}{3} = \frac{90}{3}$ $= 30$	135000	$I_{self1} = \frac{bh^3}{36}$ $= \frac{100 \times 90^3}{36}$ $= 2025000$
Rectangle	$a_2 = -20 \times 30$ $= -600$	$y_2 = \frac{30}{2} + 30$ $= 4.5$	-27000	$I_{self2} = \frac{bd^3}{12}$ $= \frac{20 \times 30^3}{12}$ $= 45000$
	$\Sigma a = 39000$		$\Sigma ay = 108000$	

Distance of centroidal axis x-x from 1-1 axis

$$\bar{y} = \frac{\Sigma ay}{\Sigma a}$$

$$= \frac{108000}{3900}$$

$$\bar{y} = 27.7 \text{ cm} \qquad \text{(Ans)}$$

QPQA 14 marks questions

M.I of entire section about x-x axis

I_{x-x} = M.I of Triangle about x-x axis–M.I of Rectangle about x-x axis $[I_{self1} + a_1(y_1 - \bar{y})^2] + [I_{self2} + a_2(y_2 - \bar{y})^2]$

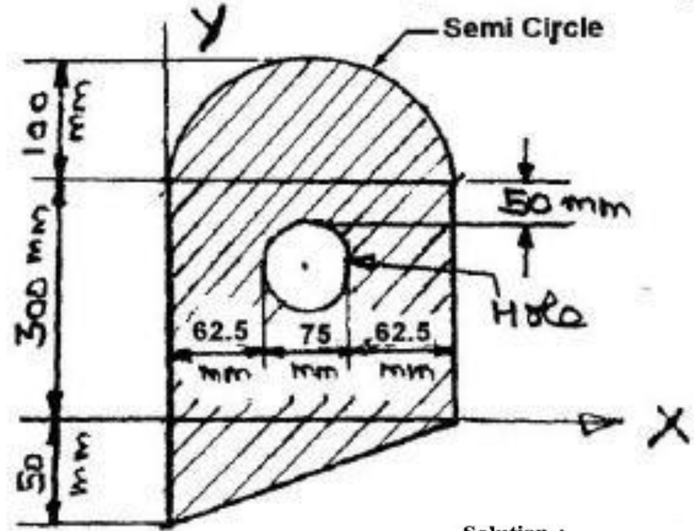
$= [2025000 + 4500(30 - 27.7)^2] - [45000 + 600(45 - 27.7)^2]$

$= 2048805 - 224574$

$I_{x-x} = 1824231\ cm^4$ (Ans)

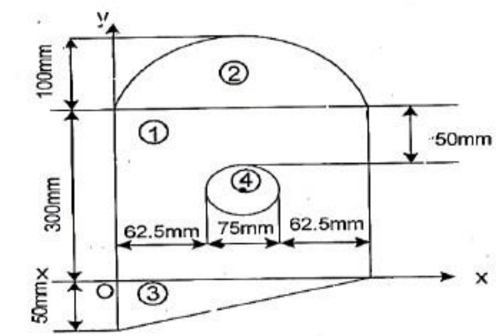
QPQA 14 marks questions

Locate the centroid of the plane area shown in figure below. **(AU DEC'11, DEC'12)**



Solution :

The given section is split up into four sections as shown in figure. Let OX and OY be the reference axis.



QPQA 14 marks questions

Component	Area 'a' (mm^2)	Centroidal Distance from O-Y axis 'x' (mm)	Centroidal Distance from O-X axis 'y' (mm)	ax (mm^2)	ay (mm^2)
Rectangle	$300 \times 200 = 60 \times 10^3$	$\frac{200}{2} = 100$	$\frac{300}{2} = 150$	6×10^6	9×10^6
Semi-circle	$\frac{\pi r^2}{2}$ $\frac{\pi \times 100^2}{2}$ $= 15.7 \times 10^3$	$\frac{200}{2} = 100$	$\frac{4r}{3\pi} + 300$ $\frac{4 \times 100}{3\pi} + 300$ $= 342.44$	1.57×10^6	5.37×10^6
Triangle	$\frac{1}{2} \times 200 \times 50$ $= 5000$	$\frac{200}{3} = 66.67$	$\frac{50}{3} = -16.67$	0.33×10^6	-83.35×10^3
Circle (–)	$-\pi r^2$ $= -\pi \times 100^2$ $= -31.41 \times 10^3$	$\frac{200}{2}$ $= 100$	$300 - \left(50 + \frac{75}{2}\right)$ $= 212.5$	-3.14×10^6	-6.67×10^6
	$\Sigma a = 49.29 \times 10^3$			$\Sigma ax = 4.76 \times 10^6$	$\Sigma ay = 7.61 \times 10^6$

QPQA 14 marks questions

Centrodial distance from O-Y axis,

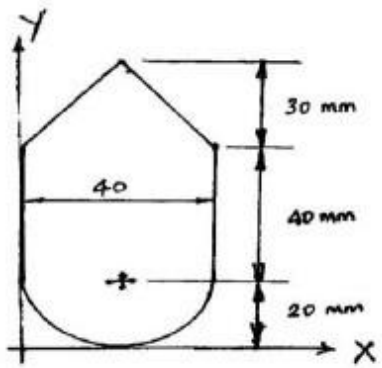
$$\bar{x} = \frac{\sum ax}{\sum a} = \frac{4.76 \times 10^6}{49.29 \times 10^3} = 96.57\,\text{mm}$$

Centrodial distance from O-X axis,

$$\bar{y} = \frac{\sum ay}{\sum a} = \frac{7.61 \times 10^6}{49.29 \times 10^3} = 154.39\,\text{mm}$$

QPQA 14 marks questions

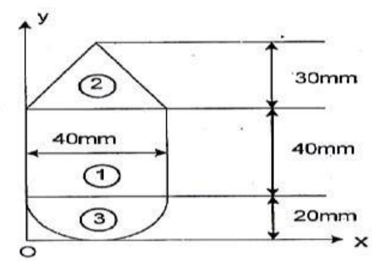
Figure shows a composite area. **(AU DEC'11, JUN' 10)**



Find the moments of inertia (second moments of area) about both the centroidal axes.

Solution :

The given section is split into three sections as shown in the figure let Ox and Oy be the reference axis.



QPQA 14 marks questions

Centrodial distance from O-y axis,

$$\bar{x} = \frac{\sum ax}{\sum a} = \frac{56.56 \times 10^3}{2828.32} = 20mm$$

Centrodial distance from O-Y axis,

$$\bar{y} = \frac{\sum ay}{\sum a} = \frac{77.23 \times 10^3}{2828.32} = 27.3mm$$

To find moment of Inertia about X-X axis:

I_{xx} = M.I of Rectanlge (1) about x-x axis + M.I of Trianlge (2) about x-x axis + M.I of Semicircle (3) about x-x axis

$$= [Iself_1 + a_1(y_1 - \bar{y})^2] + [Iself_2 + a_2(y_2 - \bar{y})^2] + [Iself_3 + a_3(y_3 - \bar{y})^2]$$

$$= [0.2136 \times 10^6 + 1600(40 - 27.3)^2] + [30 \times 10^3 + 600(10 - 27.3)^2] + [17.6 \times 10^3 + 628.32(11.51 - 27.3)^2]$$

$$= 471.664 \times 10^3 + 209.574 \times 10^3 + 174.255 \times 10^3$$

$I_{xx} = 855.49 \times 10^3\ mm^4$ **(Ans)**

To find moment of Inertia about Y-Y axis

I_{yy} = M.I of Rectanlge (1) about y-y axis + M.I of Trianlge (2) about y-y axis + M.I of Semicircle (3) about y-y axis

QPQA 14 marks questions

Component	Area 'a' (mm^2)	Centroidal Distance from O-Y 'x' (mm)	Centroidal Distance from O-X 'y' (mm)	ax (mm^3)	ay (mm^3)	Itself about x-x axis mm^4	Itself about y-y axis mm^4
(1) Rectangle	40×40 $= 1600$	$\frac{40}{2} = 20$	$\frac{40}{2} = +20$ $= 4C$	32×10^3	64×10^3	$\frac{bd^3}{12}$ $= \frac{40 \times 40^3}{12}$ $= 0.213 \times 10^6$	$\frac{bd^3}{12}$ $= \frac{40 \times 40^3}{12}$ $= 0.213 \times 10^6$
(2) Triangle	$\frac{1}{2} \times 40 \times 30$ $= 600$	$\frac{40}{2} = 20$	$\frac{30}{3} = 10$	12×10^3	6×10^3	$\frac{hb^3}{36}$ $= \frac{40 \times 30^3}{36}$ $= 30 \times 10^3$	$\frac{hb^3}{36}$ $= \frac{30 \times 40^3}{48}$ $= 40 \times 10^3$
(3) Semi circ'-	$\frac{\pi r^2}{2}$ $= \frac{\pi \times 20^2}{2}$ $= 628.32$	$\frac{40}{2} = 20$	$20 - \frac{4r}{3\pi}$ $= 20 - \frac{4 \times 20}{3\pi}$ $= 11.51$	12.56×10^3	7.23×10^3	$0.11 r^4$ $= 0.11 \times 20^4$ $= 17.6 \times 10^3$	$\frac{\pi D^4}{128}$ $= \frac{\pi (40)^4}{128}$ $= 62.83 \times 10^3$
	$\Sigma a = 2828.32$			$\Sigma ax = 56.56 \times 10^3$	$\Sigma ay = 77.23 \times 10^3$		

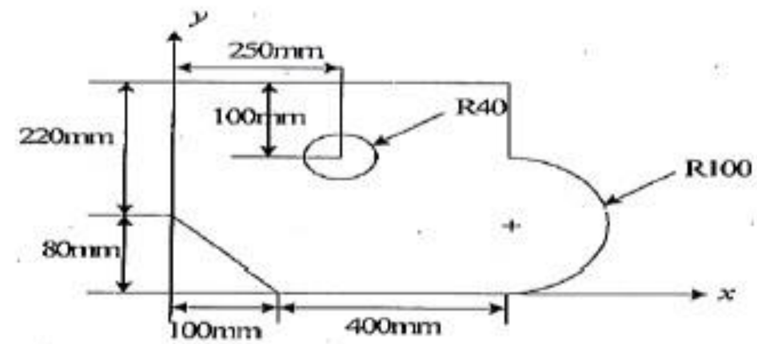
$$= [I_{self_1} + a_1 (x_1 - \bar{x})^2] + [I_{self_2} + a_2 (x_2 - \bar{x})^2] + [I_{self_3} + a_3 (x_3 - \bar{x})^2]$$

$$= [0.213 \times 10^6 + 1600(20-20)^2] + [40 \times 10^3 + 600(20-20)^2] + [62.83 \times 10^3 + 628.32(20-20)^2]$$

$$= 0.213 \times 10^6 + 40 \times 10^3 + 62.83 \times 10^3$$

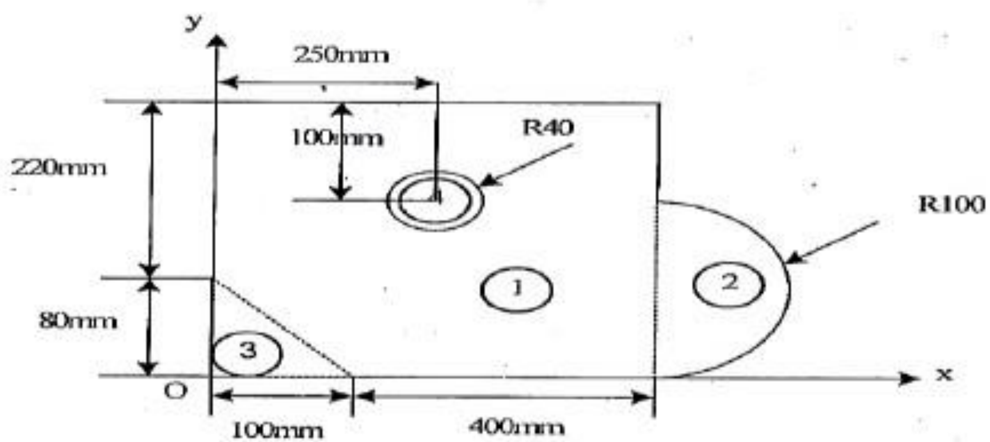
$$I_{yy} = 315.83 \times 10^3 mm^4 \qquad \textbf{(Ans)}$$

Locate the centroid of the plane area shown in figure below **(AU MAY'11)**



Solution :

The given section is split up into four sections as shown in figure. Let OX and OY be the reference axis.



QPQA 14 marks questions

Component	Area 'a' (mm^2)	Centroidal Distance from O-Y axis 'x' (mm)	Centroidal Distance from O-X axis 'y' (mm)	ax (mm^2)	ay (mm^2)
Rectangle	$300 \times 500 = 150 \times 10^3$	$\frac{500}{2} = 250$	$\frac{300}{2} = 150$	37.5×10^6	22.5×10^6
Semi-circle	$\frac{\pi r^2}{2}$ $\frac{\pi \times 100^2}{2}$ $= 15.7 \times 10^3$	$\frac{4r}{3\pi} + 500$ $\frac{4 \times 100}{3\pi} + 500$ $= 542.44$	100	8.5×10^6	1.57×10^6
Triangle (–)	$\frac{1}{2} \times 100 \times 80 = -4 \times 10^3$	$\frac{100}{3} = 33.34$	$\frac{80}{3} = 26.67$	-0.133×10^6	-0.106×10^3
Circle (–)	$-\pi r^2 = -\pi(40)^2 = -5.02 \times 10^3$	250	300–100 =200	-1.255×10^6	-1.004×10^6
	$\Sigma a = 156.68 \times 10^3$			$\Sigma ax = 82.5 \times 10^6$	$\Sigma ay = 22.96 \times 10^6$

Distance of centroid from OY axis, $\bar{x} = \frac{\Sigma ax}{\Sigma a}$

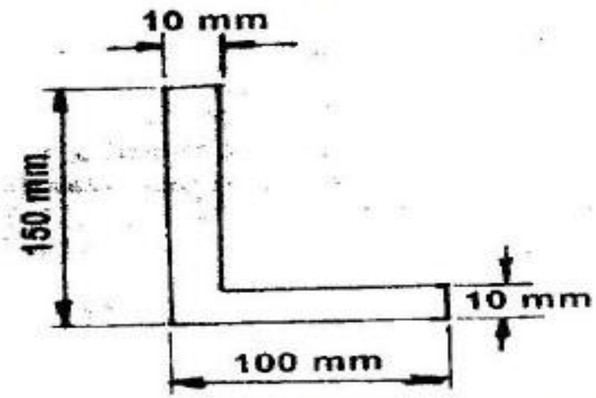
QPQA 14 marks questions

$$= \frac{44.61\times10^6}{156.68\times10^3} = 284.72\text{mm} \qquad \text{(Ans)}$$

Distance of centroid from OX axis, $\bar{y} = \frac{\sum ay}{\sum a}$

$$= \frac{22.96\times10^6}{156.68\times10^3} = 146.54\text{mm} \qquad \text{(Ans)}$$

An area in the form of L section is shown in figure below **(AU MAY'11, DEC'12)**



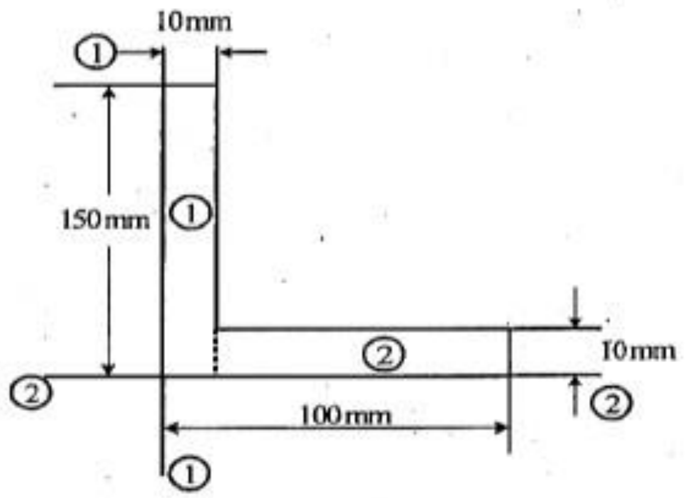
Find the moments of Inertia I_{xx}, I_{yy}, and I_{xy} about its centroidal axes. Also determine the principal moments of inertia.

QPQA 14 marks questions

Find the moments of Inertia I_{xx}, I_{yy}, and I_{xy} about its centroidal axes. Also determine the principal moments of inertia.

Solution:

The given section is split up into two rectangles as shown in figure. Let 1-1 and 2-2 be the reference axis. The calculations are shown in the table



QPQA 14 marks questions

Component	Area 'a' (mm^2)	Centroidal Distance from O-Y axis 'x' (mm)	Centroidal Distance from O-X axis 'y' (mm)	ax (mm^2)	ay (mm^2)	I_{self} about x-x axis mm^4	I_{self} about y-y axis mm^4
Rectangle (1)	150×10 =1500	$\frac{10}{2}=5$	$\frac{150}{2}=75$	7500	112.5 ×10³	$\frac{bd^3}{12}$ $\frac{10\times150^3}{12}$ $=2.81\times10^6$	$\frac{db^3}{12}$ $\frac{150\times10^3}{12}$ $=12.5\times10^3$
Rectangle (2)	90×10 =900	$10+\frac{90}{2}$ =55	$\frac{10}{2}=5$	49.5 ×10³	4500	$\frac{bd^3}{12}$ $\frac{90\times10^3}{12}$ $=7.5\times10^3$	$\frac{db^3}{12}$ $\frac{10\times90^3}{12}$ $=60.75\times10^3$
	Σa = 2400			Σax= 57×10³	Σay = 117×10³		

1. To find centroidal distance :

Distance of centroidal axis y-y from 1-1 axis,

$$\overline{x}=\frac{\Sigma ax}{\Sigma a}=\frac{57\times10^3}{2400}=23.75mm$$

QPQA 14 marks questions

1. To find centroidal distance :

Distance of centroidal axis y-y from 1-1 axis,

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{57 \times 10^3}{2400} = 23.75\text{mm}$$

Distance of centroidal axis x-x from 2-2 axis,

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{117 \times 10^3}{2400} = 48.75\text{mm}$$

2. To find moment of inertia about centroidal x-x axis (I_{xx})

M.I. of given section about centroidal x-x axis is

I_{xx} = M.I. of rectangle (1) about x-x axis + M.I of rectanlge (2) about x-x axis.

$$= [I_{self1} + a_1(y_1 - \bar{y})^2] + [I_{self2} + a_2(y_2 - \bar{y})^2]$$

QPQA 14 marks questions

$=[2.81\times10^6+1500(75-48.75)^2]+[7.5\times10^3+900(5-48.75)^2]$

$= 3.84\times10^6+1.73\times10^6$

$I_{xx} = 5.57\times10^6 mm^4$ **(Ans)**

3. To find moment of inertial about centroidal y-y axis (I_{yy})

I_{yy} = M.I. of rectangle (1) about y-y axis + M.I of rectanlge (2) about y-y axis.

$= [I_{self1}+a_1(x_1-\bar{x})^2]+[I_{self2}+a_2(x_2-\bar{x})^2]$

$=[12.5\times10^3+1500(5-23.75)^2][60.7.5\times10^3+900(55-23.75)^2]$

$= 0.539\times10^6+1.486\times10^6$

$I_{yy} = 2.025\times10^6 mm^4$ **(Ans)**

QPQA 14 marks questions

4) To find product of inertia (I_{XY}) :-

Product of inertia of given section is

$$I_{XY} = [I_{x_1'y_1'} + a_1 x_1 y_1] + [I_{x_2'y_2'} + a_2 x_2 y_2]$$

where,

x_1', x_2', y_1' and y_2' are the axes of symmetry.

$\therefore\ I_{x_1'y_1'} = 0$, $\quad \therefore\ I_{x_2'y_2'} = 0$,

$$\therefore\ I_{xy} = [0 + 1500\times5\times75] + [0+900\times55\times5]$$

$$= 0.5625\times10^6 + 0.2475\times10^6$$

$$\mathbf{I_{xy} = 8.1\times10^5\ mm^4} \qquad \textbf{(Ans)}$$

QPQA 14 marks questions

5) To find principal moment of inertia:-

The principal moments of inertia is given by

$$I_{max,\ min} = \frac{I_{xx} + I_{yy}}{2} \pm \sqrt{\left(\frac{I_{xx} - I_{yy}}{2}\right)^2 + I_{xy}^2}$$

$$= \frac{5.57\times10^6 + 2.025\times10^6}{2} \pm \sqrt{\left(\frac{5.57\times10^6 - 2.025\times10^6}{2}\right) + (8.1\times10^5)^2}$$

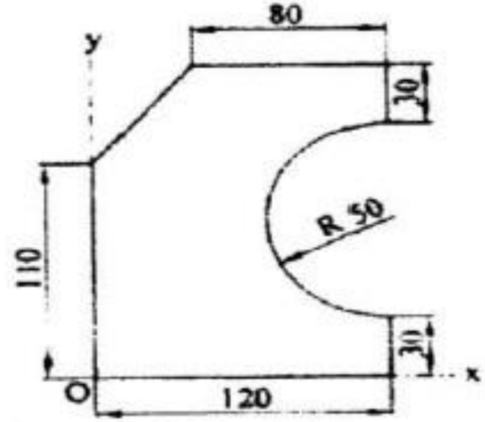
$$= 3.797\times10^6 \pm 1.948\times10^6$$

$$I_{max} = 3.797\times10^6 + 1.948\times10^6 = 5.745\times10^6\ mm^4$$

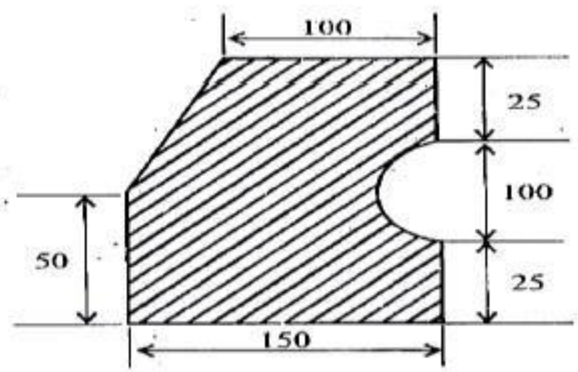
$$I_{min} = 3.797\times10^6 - 1.948\times10^6 = 1.849\times10^6\ mm^4\ \text{(Ans)}$$

QPQA 14 marks questions

. Locate the centroid of the area shown in figure below. The dimensions are in mm. **(AU JUN'10,DEC 11)**

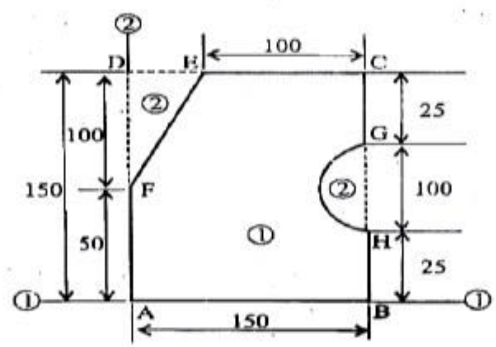


Solution :



The given section is not symmetrical about any axis Hence, we have to find the values of $\bar{x}$ and $\bar{y}$. Split up the area into square, triangle and semi-circle as in fig . The area is obtained by adding square and subtracting both the triangle and semi - circle.

The calculations are shown in the following table.



QPQA 14 marks questions

S No.	Component	Area 'a' (mm²)	Distance of centroid from 2-2 'x' (mm)	Distance of centroid from 1-1 'y' (mm)	ax (mm³)	ay (mm³)
1.	Square ABCD	150x150 =22.5x10³	$\frac{150}{2}$ = 75	$\frac{150}{2}$ = 75	1687.5x10³	1687.5x10³
2.	Semi Circle	$\frac{\pi r^2}{2}$ $= \frac{-\pi \times 50^2}{2}$ = -3.927x10³	$150 - \frac{4r}{3\pi}$ $= 150 - \frac{4\times 50}{3\pi}$ = 128.78	$\frac{100}{2} + 25$ = 75	-505.71x10³	-294.525x10³
3.	Triangle ADE	$-\frac{1}{2}bh$ $= \frac{1}{2}\times 50\times 100$ = -2.5x10³	$\frac{b}{3}$ $= \frac{50}{3}$ = 16.66	$150 - \frac{h}{3}$ $150 - \frac{100}{3}$ = 116.66	-41.662x10³	-291.665x10³
		Σa = 16.073 x 10³			Σax = 1140.12x10³	Σay = 101.31x10³

Centroid distance from 2-2 axis,

$$\bar{x} = \frac{\sum ax}{\sum a}$$

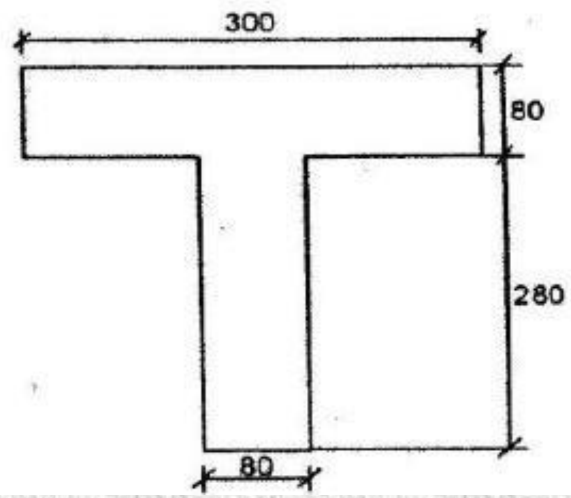
$$= \frac{1140.12 \times 10^3}{16.073 \times 10^3} \qquad \bar{x} = 70.93 \text{ mm (Ans)}$$

Centroidal distance from 1-1 axis,

$$\bar{y} = \frac{\sum ay}{\sum a}$$

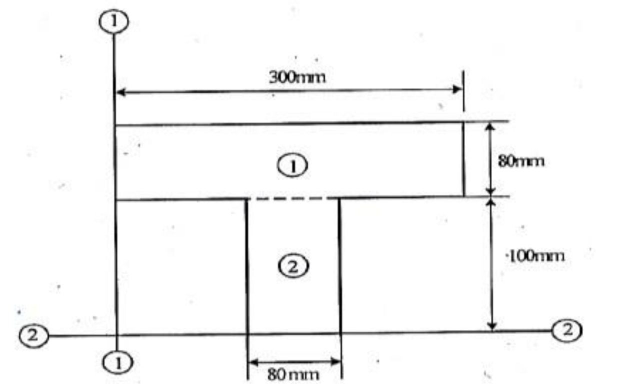
$$= \frac{1101.31 \times 10^3}{16.073 \times 10^3} \qquad \bar{y} = 68.52 \text{ mm (Ans)}$$

. Find the polar moment of inertia of a T section shown in Fig 5 about an axis passing through its centroid. Also find the radius of gyration with respect to the polar axis. (Dimensions in mm) **(AU JUN'09)**



Solution:

The given section is split up into two rectangles as shown in the figure. Let 1-1 and 2-2 be the axis of reference. The calculations are shown in the table.



QPQA 14 marks questions

Component	Area 'a' (mm^2)	Centroidal Distance from -1–1 axis 'x' (mm)	Centroidal Distance from 2–2 axis 'y' (mm)	ax (mm^3)	ay (mm^3)	I_{self} about centroidal axis x-x mm^4	I_{self} about centroidal axis y-y mm^4
(1) Rectangle	$300 \times 80 = 24 \times 10^3$	$\frac{300}{2} = 150$	$\frac{80}{2} + 100 = 140$	3.6×10^6	3.36×10^6	$\frac{bd^3}{12} = \frac{300 \times 80^3}{12} = 12.8 \times 10^6$	$\frac{db^3}{12} = \frac{80 \times 300^3}{12} = 180 \times 10^6$
(2) Rectangle	$100 \times 80 = 8 \times 10^3$	$\frac{300}{2} = 150$	$\frac{100}{2} = 50$	1.2×10^6	0.4×10^6	$\frac{bd^3}{12} = \frac{80 \times 100^3}{12} = 6.67 \times 10^6$	$\frac{db^3}{12} = \frac{100 \times 80^3}{12} = 4.26 \times 10^6$
	$\Sigma a = 32 \times 10^3$			$\Sigma ax = 4.8 \times 10^6$	$\Sigma ay = 3.76 \times 10^6$		

QPQA 14 marks questions

To find centroid:

Distance of centroidal axis y-y from 1-1 axis

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{4.8 \times 10^6}{32 \times 10^3} = 150mm$$

Distance of centroidal axis x-x from z-z axis

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{3.76 \times 10^6}{32 \times 10^3} = 117.5mm$$

To find moment of inertia about centroidal x-x axis.

M.I. of the section about horizontal centroidal x-x axis is

$I_{xx} =$ M.I. of rectangle(1) about x-x axis – M.I. of rectangle(2) about y-y axis

$$= \left[I_{self_1} + a_1(\bar{y} - y_1)^2\right] + \left[I_{self_2} + a_2(\bar{y} - y_2)^2\right]$$

$$= [1.28\times10^6 + 24\times10^3(117.5-140)^2] + [6.67\times10^6 + 8\times10^3(117.5-50)^2]$$

$$= 24.95\times10^6 + 43.12\times10^6$$

$$I_{xx} = 68.07\times10^6 \text{ mm}^4$$

To find moment of inertia about centroidal y-y axis.

M.I. of section of about vertical centroidal y-y axis is

I_{yy} = M.I. of rectangle(1) about y-y axis – M.I. of rectangle(2) about y-y axis

$$= \left[I_{self1} + a_1(\bar{x} - x_1)^2 \right] + \left[I_{self_2} + a_2(\bar{x} - x_2)^2 \right]$$

$$= [180\times10^6 + 24\times10^3(150-150)] + [4.26\times10^6 + 8\times10^3(150-150)^2]$$

$$= [180\times10^6 + 4.26\times10^6]$$

$$\mathbf{I_{yy} = 184.26\times10^6 \ mm^4}$$

To find polar moment of inertia:

Polar moment of inertia of given T-section is

$$J = I_{xx} + I_{yy}$$

$$J = 68.07\times10^6 + 184.26\times10^6 = 252.38 \ mm^4$$

To find radius of gyration with respect to polar axis:

Radius of gyration about x-x axis

$$K_{x-x} = \sqrt{\frac{I_{xx}}{\Sigma a}} = \sqrt{\frac{68.07 \times 10^6}{32 \times 10^3}} = 46.12 \text{ mm}$$

Radius of gyration about y-y axis.

$$K_{y-y} = \sqrt{\frac{I_{yy}}{\Sigma a}} = \sqrt{\frac{184.26 \times 10^6}{32 \times 10^3}} = 75.88 \text{ mm}$$

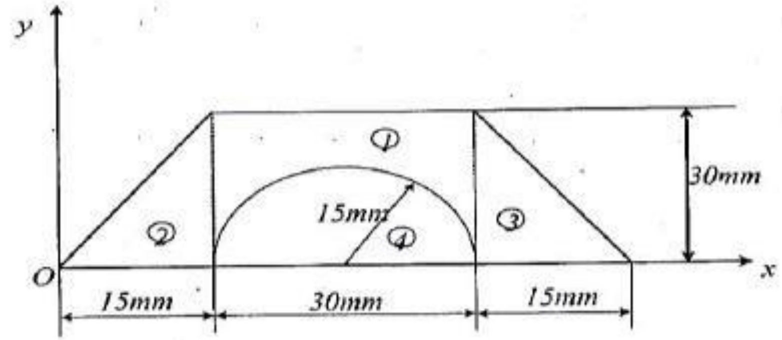
Radius of gyration about polar axis is

$$K_p = \sqrt{K_{xx}^2 + K_{yy}^2}$$

$$= \sqrt{(46.12)^2 + (75.88)^2}$$

$$\mathbf{K_p = 88.79 \text{ mm}} \qquad \textbf{(Ans).}$$

Calculate the centroidal moment of inertia of the shaded area shown in figure below. **(AU DEC'09,JUN'12)**



Solution:

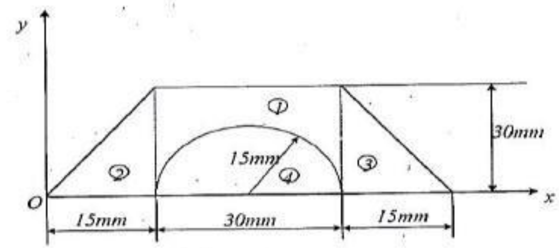
The given area is split up into various sections as shown in figure. Let OX and OY be the reference axis.

The calculations are shown in the table.

i. To find centroid:

Distance of Centroidal axis YY from O - Y is

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{29.89 \times 10^3}{996.57} = 30mm$$



QPQA 14 marks questions

Component	Area -a mm^2	Centroidal distance from 0-y axis 'x' (mm)	Centroidal distance from 0-x axis 'y' (mm)	ax mm^3	ay mm^3	Iseif about x-x axis mm^4	Iseif about y-y axis mm^4
Rectangle (1)	$30\times30 = 900$	$\frac{30}{2}+5=30$	$\frac{30}{2}=15$	27×10^3	13.5×10^3	$\frac{bd^3}{12}=\frac{30\times30^3}{12} = 67.5\times10^3$	$\frac{db^3}{12}=\frac{30\times30^3}{12} = 67.5\times10^3$
Triangle (2)	$\frac{1}{2}\times15\times30 = 225$	$\frac{2}{3}\times6= \frac{2}{3}\times15=10$	$\frac{4}{3}=\frac{30}{3} = 10$	2.25×10^3	2.25×10^3	$\frac{bh^3}{36}=\frac{15\times30^3}{36} = 11,250$	$\frac{ab^3}{36}=\frac{30\times15^3}{36} = 2812.5$
Triangle (3)	$\frac{1}{2}\times15\times30 = 225$	$\frac{15}{3}+30+15 = 50$	$\frac{30}{3}=10$	11.25×10^3	2.25×10^3	$\frac{15\times30^3}{36} = 11,250$	$\frac{30\times15^3}{36} = 2812.5$
Semicircle (4) (–)	$\frac{51\times15^2}{2} = -(353.43)$	$\frac{30}{2}+15 = 30$	$\frac{4\gamma}{3\pi}=\frac{4\times15}{3\pi} = 6.366$	(10.602×10^3)	(2.25×10^3)	$0.11\gamma^4 = 0.11\times15^4 = 5.56\times10^3$	$\frac{\pi d^4}{128}=\frac{\pi\times30^4}{128} = 19.88\times10^3$
	$\Sigma a = 996.57$			$\Sigma ax = 29.89\times10^3$	$\Sigma ay = 15.75\times10^3$		

QPQA 14 marks questions

Distance of Centroidal axis XX from O - X is

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{15.75\times10^3}{996.57} = 15.8 \text{ mm}$$

To find momentof Inertia about X - X axis:

M.I of entire section about X - X axis,

I_{X-X} = M.I of Rectangle (1) about x axis+ M.I of triangle (2) about x-x axis + M.I of triangle (3) about x-x axis – M.I of semi circle (4) about x-x axis

$$= [Iself_1 + a_1(\bar{y} - y_1)^2] + [Iself_2 + a_2(\bar{y} - y_2)^2]$$

$$+ [Iself_3 + a_3(\bar{y} - y_3)^2] + [Iself_4 + a_4(\bar{y} - y_4)^2]$$

$$= [67.5 \times 10^3 + 900(15.8-15)^2] + [11250 + 225(15.8-10)^2] + [11250+225(15.8-10)^2] - [5.56\times10^3+353.43(15.8 - 6.36)^2]$$

$$= 68076 + 18819 + 18819 - 37055.42$$

$$I_{XX} = 68.65 \times 10^3 \text{ mm}^4 \qquad \text{(Ans)}$$

Two trains A and B leave the same station on parallel lines. A starts with a uniform acceleration of $0.15 m/s^2$ and attains the speed of 24 km/hour after which its speed remains constant. B leaves 40 seconds later with uniform acceleration of $0.30 m/s^2$ to attain a maximum of 48 km/hour, its speed also becomes constant thereafter. When will B overtake A. **(AU,Dec'11,JUN 12)**

Solution :

Consider the motion of Train A:

Initial velocity, $u = 0$

Final velocity, $V = 24$ km/hr

$$= \frac{24 \times 1000}{3600} = 6.67 m/s^2$$

Acceleration, $a = 0.15 m/s^2$

T= time taken when the train B will overtake the train A from its start.

t_A = time taken by train A to attain a speed of $6.67 m/s^2$

$$V = u + a\, t_A$$

$$6.67 = 0 + 0.15\, t_A$$

$$t_A = 44.67 \text{ sec.}$$

QPQA 14 marks questions

$$S_1 = u\,t_A + \frac{1}{2}\,a\,t_A^2$$

$$S_1 = 0 + \frac{1}{2}\,0.15\times(44.67)^2$$

$$S_1 = 150m$$

Since the train B leaves 40 seconds later, so that the train A has travelled (T+40) sec.

$\therefore$ Distance travelled by train A in (T+60) sec,

$$S_A = S_1 + V[(T+60)-t_A]$$

$$S_A = 150+6.67[(T+60)-44.67] \quad ......(1)$$

Consider the motion of Train B

Initial velocity, $u = 0$

Final velocity, $V = 48$ km/hr

$$= \frac{48\times1000}{3600} = 13.34 \text{ m/s}$$

Acceleration, $a = 0.30$ m/s²

t_B = time taken by train B to attain a speed of 13.34 m/s.

$$V = u + a\,t_B$$

$$13.34 = 0 + 0.3t_B$$

$$t_B = 44.47 \text{ sec.}$$

Distance travelled by train B in 44.47 sec.

$$S_2 = ut_B + a\,t_B^2$$

$$S_2 = 0 + \frac{1}{2}\,0.3\times(44.47)^2$$

$$S_2 = 296.63m$$

$\therefore$ Distance travelled by train B in T seconds is

$$S_B = S_2 + V(T-t_B)$$

QPQA 14 marks questions

$$S_B = 296.63+13.34(T-44.47)-44.67] \; .....(2)$$

At the instent, when train B overtake trains will be equal.

Hence

$$S_A = S_B$$

$$150 + 6.67\;[(T+60)-44.67] = 296.63+13.34\;(T-44.47)$$

$$150 + 6.67T+400.2-297.94 = 296.63+13.34T-593.22$$

$$6.67T+252.26 = 13.34T-296.59$$

$$6.67T = 548.85$$

$$\mathbf{T = 82.28 \text{ seconds}} \qquad \textbf{(Ans)}$$

Car A accelerates uniformly from rest on a straight level road. Car B starting from the same point 6 seconds later with zero initial velocity accelerates at $6 m/s^2$. It overtakes the car A at 400m from the starting point. What is the acceleration of the car A? **(AU, Apr'11)**

Given :

Initial velocity of car A, $u_A = 0$

Initial velocity of car B, $u_B = 0$

acceleration of car B, $a_B = 6 m/s^2$

Distance travelled by car A and car B, $S_A = S_B = 400m$

To Find :

Acceleration to car A, $a_a = ?$

Solution :

Let 't_A' be the time taken by car 'A'.

Since the car 'B' starts 6 seconds later, the time taken by car B is, $t_B = t_A - 6$

Consider motion of car 'A'

$$S_A = u_A t_A + \frac{1}{2} a_A t_A^2$$

QPQA 14 marks questions

$$a_A t_A^2 = 800 \qquad \text{..... (1)}$$

Consider motion of car 'B'

$$S_B = u_B t_B + \frac{1}{2} a_B t_B^2$$

$$400 = 0 + \frac{1}{2} \times 6(t_A - 6)^2$$

$$\frac{800}{6} = t_A^2 + 36 - 12\, t_A$$

$$t_A^2 - 12 t_A - 97.33 = 0$$

Solving we get, $t_A = 17.54$ sec.

Substituting $t_A = 17.54$ sec. in eqn. (1)

$$a_A (17.54)^2 = 800$$

$$a_A = 2.6 \text{ m/s}^2 \qquad \text{(Ans)}$$

QPQA 14 marks questions

A stone is dropped into a well . The sound of the splash is heard 3.63 seconds later.How far below the ground is the surface of water in the well? . Assume the velocity of sound as 331m/s. **(AU, Apr'11, Dec '12)**

Given:

Velocity of sound, $v = 350$ m/s

Initial velocity, $u = 0$

Solution:

Let t = time taken by stone to reach bottom of well

Depth of well is

$$h = ut + \frac{1}{2}gt^2$$

$$= 0 + \frac{1}{2} \times 9.81 \times t^2$$

$$h = 4.9\ t^2 \qquad \text{........ (1)}$$

We know,

Time taken by sound to reach the top

$$= \frac{\text{Depth of well}}{\text{Velocity of sound}}$$

$$= \frac{h}{350}$$

$$= \frac{4.9\ t^2}{350}$$

QPQA 14 marks questions

It is given that,

Total time taken = 3 seconds

Total time = time taken by stone to reach bottom of well + time taken by sound to reach the top of well.

$$3 = t + \frac{4.9\, t^2}{350}$$

$$1050 = 350t + 4.9\, t^2$$

$$4.9\, t^2 + 350t - 1050 = 0$$

$$t = \frac{-350 \pm \sqrt{(350)^2 - 4 \times 4.9(-1050)}}{2 \times 4.9}$$

$$= \frac{-350 \pm 378.26}{9.8}$$

$$t = 2.9 \text{ seconds}$$

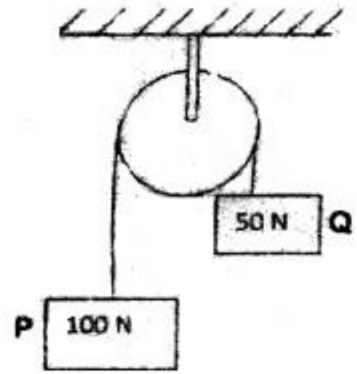
Substituting the value of 't' in equation (1) we get,

$$h = 4.9\,(2.9)^2$$

$$h = 41.21\text{m} \qquad \text{(Ans)}$$

QPQA 14 marks questions

Block P of weight 100N and block Q of weight 50N are connected by a rope that passes over a smooth pulley as shown in figure. Find the acceleration of the blocks and the tension in the rope, when the system is released from rest. Neglect the mass of the pulley. **(AU, Apr'11, Dec'12)**



Given

Weight of block P, W_P=100N

Weight of block, Q, W_Q=50N

To find

1. Acceleration of blocks, a = ?
2. Tension in the rope, T = ?

Solution

Let 'T' be the tension in the string.

Since the weight of block 'P' is larger, it will moves downward and the block 'Q' moves upward.

Considering the motion of block 'Q':-

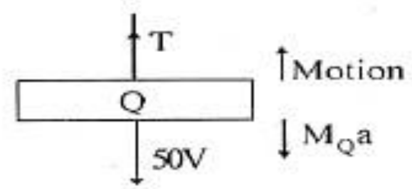
The various forces acting on block 'Q' is shown in figure.

Resolving forces vertically,

$$\Sigma F_y = 0$$

$$T - 50 - m_Q a = 0$$

$$T - 50 - \frac{50}{9.81} \times a = 0$$



$$T = 50 + 5.09a \qquad \text{......(1)}$$

Considering the motion of block 'P'

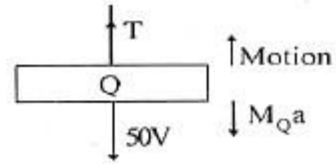
The various forces acting on block 'P' is shown in figure.

Resolving forces vertically,

QPQA 14 marks questions

$\Sigma F_y = 0$

$$T - 50 - m_Q a = 0$$



$$T - 50 - \frac{50}{9.81} \times a = 0$$

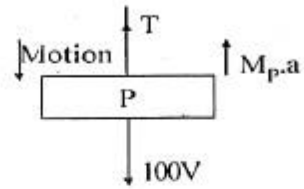
$$T = 50 + 5.09a \quad \text{......(1)}$$

Considering the motion of block 'P'

The various forces acting on block 'P' is shown in figure.

Resolving forces vertically,

$\Sigma F_y = 0$



$$T + m_{p.}a - 100 = 0$$

$$T + \frac{100}{9.81} \times a - 100 = 0$$

$$T = 100 - 10.19a \quad \text{......(2)}$$

Solving equations (1) & (2) we get

$$50 + 5.09a = 100 - 10.19a$$

$$15.28a = 50$$

$$\mathbf{a = 3.27\ m/s^2} \quad \textbf{(Ans)}$$

Put a = 3.27 in equation (2), we get

$$T = 100 - 10.19\,(3.27)$$

$$\mathbf{T = 66.67\ N} \quad \textbf{(Ans)}$$

A 2000 kg automobiles is driven down a 5° inclined plane at a speed of 100km/h when the brakes are applied causing a constant total braking force (applied by the road on the tires) of 7KN.Determine the distance travelled by automobile as it comes to a stop.

(AU, Apr'11, Dec'12)

Given:

Mass of the automobile, m =200kg

$\therefore$ W = 2000×9.81=19620N =19.62kN

Initial velocity of car, u = 100km/hr

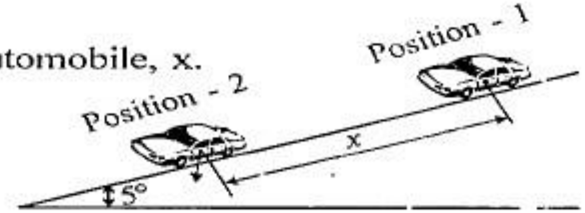
$$= \frac{100 \times 1000}{3600} = 27.78 \text{ m/s}$$

Final velocity of car, v = 0

Total breaking force, F = 7 kN

To find:

Distance travelled by automobile, x.



Solution:

To find work done:

QPQA 14 marks questions

Total work done

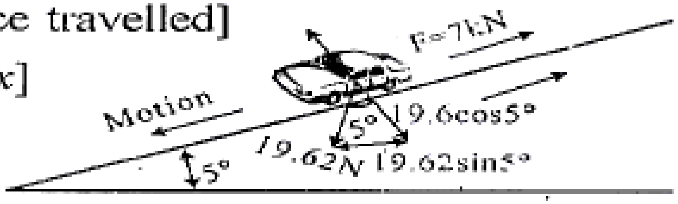
= [Force component parallel to plane×distance travelled]

= [Breaking force×distance travelled]

= [19.62 sin5°×x] – [7×x]

= 1.709x – 7x

= –5.291x



To find change in kinetic energy :

Initial Kinetic energy of automobile at position 1,

$$= \frac{1}{2} mu^2$$

$$= \frac{1}{2} \times 2000 \times 27.78^2$$

$$= 77.1.72 \times 10^3 \text{ Joules.}$$

Final Kinetic energy of auotmobile, at position 2,

$$= \frac{1}{2} mu^2 = \frac{1}{2} \times 2000 \times 0$$

= 0 Joules

QPQA 14 marks questions

Change in Kinetic Energy = Final Kinetic Energy−Initial Kinetic Energy

$= 0 - 771.72 \times 10^3$

$= -771.72$ KJ.

by work energy principle

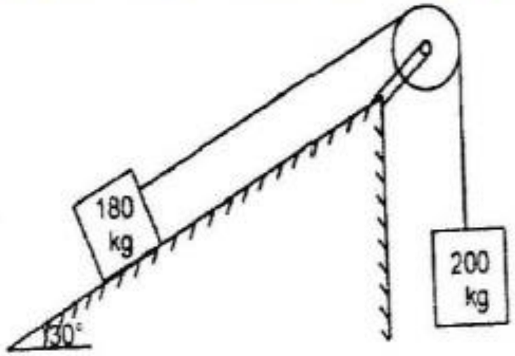
Total workdone = change in Kinetic energy

$-5.29/x = -771.72$

$x = 145.8$ **m** **(Ans)**

QPQA 14 marks questions

A block and pulley system is shown in fig below. The coefficient of kinetic friction between the block and the plane is 0.25. The pulley is frictionless. Find the acceleration of the blocks and the tension in the string when the system is just released. Also find the time required for 200kg block to come down by 2m. **(AU, Jun'09,DEC 11)**



Given:

Weight, $m_1 = 180\,kg$

$w_1 = 180 \times 9.81 = 1765.8\,N$

$m_2 = 200\,kg$

$w_2 = 180 \times 9.81 = 1765.8\,N$

Co-efficient of friction, $K = 0.25$

distance, $s = 2m$

QPQA 14 marks questions

To Find:

1. Acceleration of blocks, a = ?
2. Tension is string, T = ?
3. Time required, t = ?

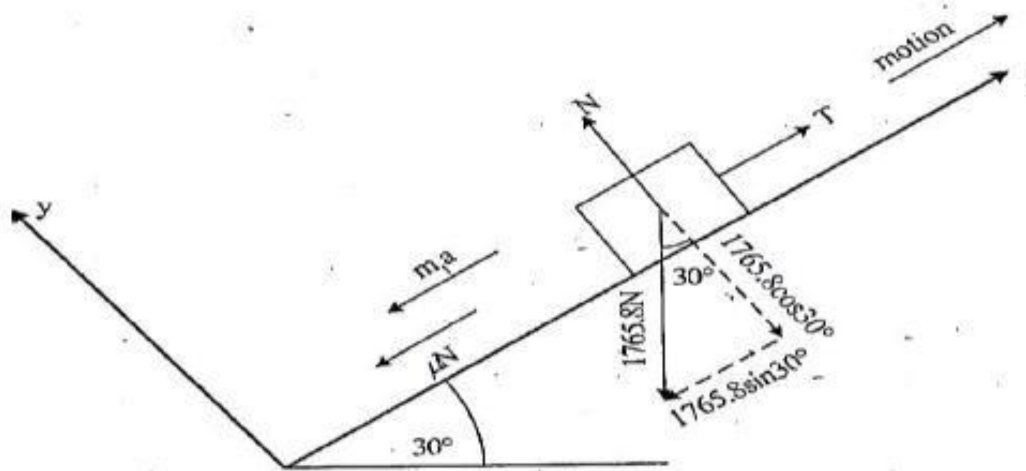
Solution:

1) To find acceleration of blocks:

Let 'T' be the tension in the string.

Considering the motion of 180 kg block

The forces acting on 180 kg block is shown in the figure.



Resolving forces horizontally,

$$\Sigma F_x = 0,$$

$$T - m_1 a - \mu N - 1765.8 \sin 30^\circ = 0$$

$$T - 180a - 0.25\,N - 882.9 = 0 \qquad .....(1)$$

Resolving forces vertically,

$\Sigma F_y = 0,$

$N - 1765.8 \cos 30^\circ$;

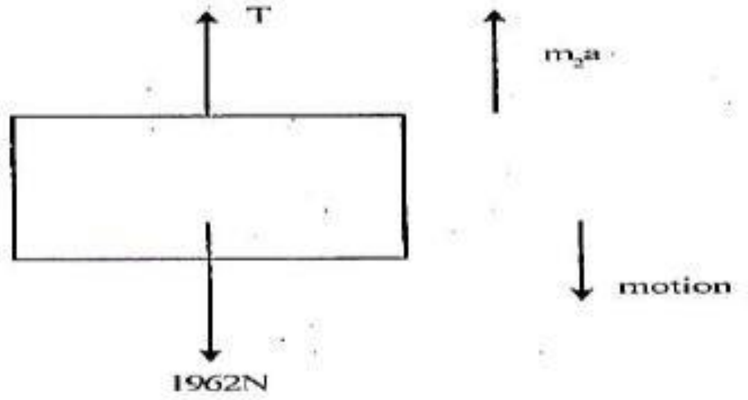
$N = 1529.22$ N

Put $N = 1529.22$ in eqn. (1)

$T - 180a - 0.25\,(1529.22) - 882.9 = 0$

$T - 180a = 1265.205$ .....(2)

Considering motion of 200 kg block, The forces acting on 200 kg block is shown in figure.



Resolving forces vertically

$T + m_2a - 1962 = 0$

$T + 200\,a = 1962$ ....(3)

Solving eqns. (2) & (3) we get

$a = 1.833\ m/s^2$

QPQA 14 marks questions

2) To find tension in the string

Put $a = 1.833$ in eqn. (3),

$T + 200(1.833) = 1962$

$T = 1595.4$ N (Ans)

3) To find time required.

Since the blocks starts from rest, its initial velocity, $u = 0$

using relation

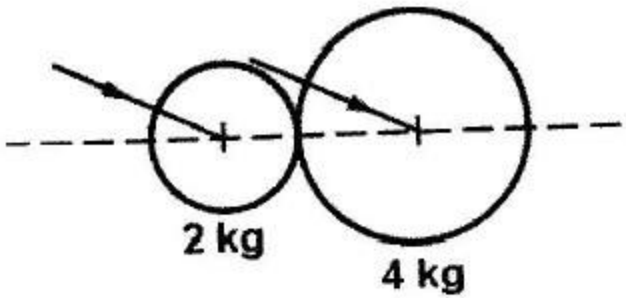
$$s = ut + \frac{1}{2}at^2$$

$$2 = 0 + \frac{1}{2} \times 9.81 \times t^2$$

$t = 0.638$ seconds (Ans)

QPQA 14 marks questions

A ball of mass 2kg, moving with a velocity of 3m/s,impinges on a ball of mass 4 kg moving with a velocity of 1m/s.The velocities of the two balls are parallel and inclined at 30°to the line of joining their centres at the instant of impact. If the coefficient of restitution is 0.5, find **(AU,Dec'09, Jun'10)**



(i) Direction, in which the 4kg ball will move after impact;

(ii) Velocity of the 4 kg ball after impact;

(iii)Direction, in which the 2kg ball will move after impact;

(iv)Velocity of the 4kg ball after impact.

QPQA 14 marks questions

Given:

Mass of first ball, $m_1 = 2Kg$

Mass of second ball, $m_2 = 4Kg$

Initial velocity of first ball, $U_1 = 3m/s$

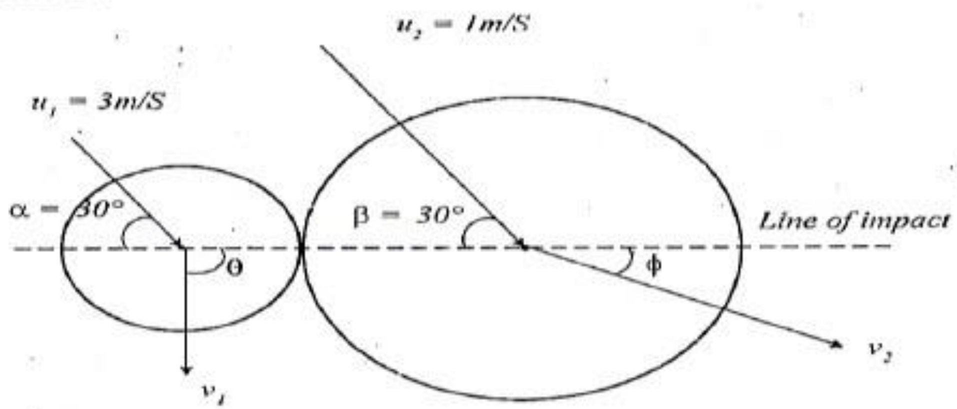
Initial velocity of second ball, $U_2 = m/s$

Angle made by first ball with line of impact, $\alpha = 30^o$

Angle made by 2^{nd} ball with line of impact, $\beta = 30^o$

Coefficient of restitution, $e = 0.5$

Solution :



QPQA 14 marks questions

Let,

Final velocity of first ball after impact $= V_1$

Final velocity of second ball after impact $= V_2$

Angle made by first ball after impact $= \theta$

Angle made by second ball after impact $= \phi$

The components of velocity of each ball perpendicular to line of impact before and after impact is same.

For Ball I,

Vertical component of initial velocity = vertical component of final velocity.

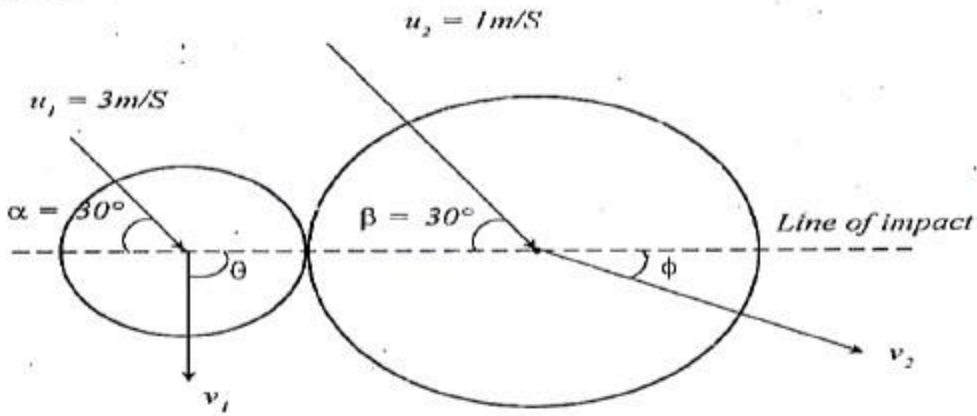
$$U_1 \sin\alpha = V_1 \sin\theta$$

$$3 \sin 30^o = V_1 \sin\theta$$

Angle made by 2nd ball with line of impact, $\beta = 30^o$

Coefficient of restitution, $e = 0.5$

Solution :



Let,

Final velocity of first ball after impact $= V_1$

Final velocity of second ball after impact $= V_2$

Angle made by first ball after impact $= \theta$

Angle made by second ball after impact $= \phi$

The components of velocity of each ball perpendicular to line of impact before and after impact is same.

For Ball I,

Vertical component of initial velocity = vertical component of final velocity.

QPQA 14 marks questions

$$U_2 \sin\beta = V_2 \sin\phi$$

$$1 \sin 30^o = V_2 \sin\phi$$

$$V_2 \sin\phi = 0.5 \quad .....(2)$$

By law of conservation of momentum,

Momentum before impact = moment after impact.

$$m_1 u_1 \cos\alpha + m_2 u_2 \cos\beta = m_1 v_1 \cos\theta + m_2 v_2 \cos\phi$$

$$2 \times 3 \times \cos 30^o + 4 \times 1 \times \cos 30^o = 2v_1 \cos\theta + 4v_2 \cos\phi.$$

$$2v_1 \cos\theta + 4v_2 \cos\phi = 8.66$$

$$v_1 \cos\theta + 2v_2 \cos\phi = 4.33 \quad ....(3)$$

Co-efficient of restitution is

$$e = \frac{v_2 \cos\phi - v_1 \cos\theta}{u_1 \cos\alpha - u_2 \cos\beta}$$

$$0.5 = \frac{v_2 \cos\phi - v_1 \cos\theta}{3 \cos 30^o - 1 \cos 30^o}$$

$$0.5\ [3 \times 0.866 - 1 \times 0.866] = v_2 \cos\phi - v_1 \cos\theta$$

$$v_2 \cos\phi - v_1 \cos\theta = 0.866 \quad ...(4)$$

Adding equations (3) & (4), we get

$$3v_2 \cos\phi = 5.196$$

$$v_2 \cos\phi = 1.732 \quad ....(5)$$

i) To find direction in which 4kg ball move after impact:

Dividing equations (2) by (5)

$$\frac{(2)}{(5)} \Rightarrow \frac{v_2 \sin\phi}{v_2 \cos\phi} = \frac{0.5}{1.732}$$

$$\tan\phi = 16.1 \quad \textbf{(Ans)}$$

ii) To find velocity of 4Kg ball after impact,

Put $\phi = 16.1$ in eqn. (2),

$$v_2 \sin 16.1 = 0.5$$

$$v_2 = 1.803 \text{ m/sec.} \quad \textbf{(Ans)}$$

iii) To find direction in which 2 Kg. ball move after impact,

substituting the values of ϕ and V_2 in eqn. (4)

$$1.803 \cos 16.1 - v_1 \cos\Theta = 0.866$$

$$v_1 \cos\Theta = 1.803 \cos 16.1 - 0.866$$

$$v_1 \cos\Theta = 0.866 \quad .....(6)$$

$$\frac{(1)}{(6)} \Rightarrow \frac{v_1 \sin\Theta}{v_1 \cos\Theta} = \frac{1.5}{0.866}$$

$$\tan\Theta = 1.732$$

$$\Theta = 60^\circ \quad \textbf{(Ans)}$$

QPQA 14 marks questions

iv) To find velocity of 2kg ball after impact.

Substitute $\theta = 60^\circ$ in eqn. (1),

$$v_1 \sin 60^\circ = 1.5$$

$$v_1 = 1.732 \text{ m/sec.} \qquad \text{(Ans)}$$

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Thank you