Heat conduction describes the process of heat transfer due to temperature differences within a material. It can occur in steady or unsteady states and in various dimensions, such as 1-D and 2-D.

1-D Heat Conduction

Steady-State:

- In steady-state heat conduction, the temperature field does not change with time. For 1-D conduction, this occurs along a single spatial direction (e.g., the x-axis).

Governing Equation:

$$\frac{d^2T}{dx^2} = 0$$

Where:

- T = temperature as a function of position
- x = spatial coordinate

Solution: The general solution is a linear temperature profile:

$$T(x) = C_1 x + C_2$$

Where C_1 and C_2 are constants determined by boundary conditions.

Unsteady-State:

- In unsteady-state heat conduction, the temperature field varies with both space and time.

Governing Equation:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

Where:

- t = time
- α = thermal diffusivity

2-D Heat Conduction

Steady-State:

- In 2-D steady-state heat conduction, the temperature does not vary with time but changes in two spatial directions (e.g., x and y).

Governing Equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

This is the **2-D Laplace equation**, which can describe steady heat conduction in materials with two-dimensional geometry.

Unsteady-State:

- In 2-D unsteady-state heat conduction, the temperature varies with time and in both x and y directions.

Governing Equation:

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

This is the **2-D heat equation**, which requires both initial and boundary conditions for a complete solution. Analytical solutions can be challenging, and numerical methods like finite difference or finite element methods are often used.

Example Applications:

- **1-D Steady-State:** Heat transfer through a flat wall or a long rod.
- **1-D Unsteady-State:** Temperature change in a rod after a sudden change in environmental conditions.
- **2-D Steady-State:** Heat conduction in a rectangular plate with different boundary conditions (e.g., constant temperature on edges).
- **2-D Unsteady-State:** Heat conduction in a sheet after a sudden temperature change on one side.