UNIT 4 Fourier Transforms
Sine and cosine Transforms

Fourier Sine Transform:

The fourier sine transform of $f(x)$ is defined by

$$F_s[s] = F_s[f(x)] = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\sin sx\,dx$$

The inverse fourier sine transform of $F_s(s)$ is given by,

$$f(x) = \sqrt{\frac{2}{\pi}}\int_0^\infty F_s(s)\sin sx\,ds$$

Fourier Cosine Transform:

The fourier cosine transform of $f(x)$ is defined by

$$F_c(s) = F_c(f(x)) = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos sx\,dx$$

The inverse fourier cosine transform of $F_c(s)$ is given by

$$f(x) = \sqrt{\frac{2}{\pi}}\int_0^\infty F_c(s)\cos sx\,ds$$

Parseval's Identity:

Sine transform:

If $F(s)$ is the Fourier transform of $f(x)$ then

$$\int_0^\infty [f(x)]^2dx = \int_0^\infty [F_s(s)]^2 ds$$

Cosine transform:

If $F(s)$ is the Fourier transform of $f(x)$, then

$$\int_0^\infty [f(x)]^2dx = \int_0^\infty [F_c(s)]^2 ds$$

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1. Find the fourier sine Transform of f(x) defined as

$$f(x) = \begin{cases} 1, & \text{if } 0 \le x < 1 \\ 0, & x > 1 \end{cases}$$

$$F_s(s) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin sx \, dx$$

$$= \sqrt{\frac{2}{\pi}} \int_0^1 \sin sx \, dx = \sqrt{\frac{2}{\pi}} \left[\frac{-\cos sx}{s} \right]_0^1$$

$$= \sqrt{\frac{2}{\pi}} \left[\frac{-\cos s + \cos 0}{s} \right] = \sqrt{\frac{2}{\pi}} \left[\frac{1 - \cos s}{s} \right]$$

2. Find the fourier sine Transform of $1/x$.

$$F_s(s) = \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{1}{x} \sin sx \, dx.$$

Put $\theta = sx \Rightarrow d\theta = s\,dx \Rightarrow \frac{d\theta}{s} = dx$

$$= \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{\sin\theta}{\theta} \, d\theta$$

$$\because \int_0^\infty \frac{\sin\theta}{\theta} \, d\theta = \frac{\pi}{2}$$

$$= \sqrt{\frac{2}{\pi}} \times \frac{\pi}{2} = \sqrt{\frac{\pi}{2}}$$

3. Find the fourier cosine transform of $2e^{-3x} + 3e^{-2x}$

$$F_c(s) = \sqrt{\frac{2}{\pi}} \int_0^\infty (2e^{-3x} + 3e^{-2x}) \cos sx \, dx$$

$$= \sqrt{\frac{2}{\pi}} \left[2\int_0^\infty e^{-3x} \cos sx \, dx + 3\int_0^\infty e^{-2x} \cos sx \, dx \right]$$

$$= \sqrt{\frac{2}{\pi}} \left[2 \left[\frac{3}{s^2+9} \right] + 3 \left[\frac{2}{s^2+4} \right] \right]$$

$$= \sqrt{\frac{2}{\pi}} \left[\frac{6}{s^2+9} + \frac{6}{s^2+4} \right]$$