UNIT 5 Z - Transforms and Difference equations
Solution of Difference Equation

Solving ~~the~~ of difference Equation :-

Formulas!

$$Z[y_n] = F(z)$$

$$Z[y_{n+1}] = zF(z) - zy_0$$

$$Z[y_{n+2}] = z^2F(z) - z^2y_0 - zy_1$$

$$Z[y_{n+3}] = z^3F(z) - z^3y_0 - z^2y_1 - zy_2.$$

where $y_0 = y(0)$

$y_1 = y(1)$

$y_2 = y(2)$

1. Solve $y_{n+2} + 4y_{n+1} + 3y_n = 2^n$. with $y_0 = 0$ and $y_1 = 1$ using z-transform.

$$y_{n+2} + 4y_{n+1} + 3y_n = 2^n.$$

Taking z-transform on both sides,

$$Z[y_{n+2}] + 4Z[y_{n+1}] + 3Z[y_n] = Z[2^n]$$

$$z^2F(z) - z^2y_0 - zy_1 + 4[zF(z) - zy_0] + 3F(z) = \frac{z}{z-2}$$

$$z^2(F(z)) - 0 - z + 4zF(z) + 3F(z) = \frac{z}{z-2}$$

$$(z^2+4z+3)F(z) = \frac{z}{z-2} + z$$

$$F(z) = \frac{z^2 + z - 2z}{(z-2)(z^2+4z+3)}$$

$$= \frac{z^2 - z}{(z-2)(z^2+4z+3)}$$

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$$F(z) = \frac{z^2 - z}{(z-2)(z+1)(z+3)}$$

$$\frac{F(z)}{z} = \frac{z-1}{(z-2)(z+1)(z+3)} \longrightarrow ①$$

$$\frac{z-1}{(z-2)(z+1)(z+3)} = \frac{A}{z-2} + \frac{B}{z+1} + \frac{C}{z+3}$$

$$= \frac{A(z+1)(z+3) + B(z-2)(z+3) + C(z-2)(z+1)}{(z-2)(z+1)(z+3)}$$

$$z - 1 = A(z+1)(z+3) + B(z-2)(z+3) + C(z-2)(z+1)$$

When $z = -1$, $-1-1 = B(-1-2)(-1+3)$

$$-2 = B(-3)(2)$$

$$-6B = -2$$

$$\boxed{B = 1/3}$$

When $z = 2$, $2-1 = A(2+1)(2+3)$

$$1 = A(3)(5)$$

$$\boxed{A = 1/15}$$

When $z = -3$, $-3-1 = C(-3-2)(-3+1)$

$$-4 = C(-5)(-2)$$

$$C = 4/10$$

$$\boxed{C = -2/5}$$

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$$① \Rightarrow \frac{F(z)}{z} = \frac{1}{15}\frac{1}{z-2} + \frac{1/3}{z+1} + \frac{-2/5}{z+3}$$

$$F(z) = \frac{1}{15}\frac{z}{z-2} + \frac{1}{3}\frac{z}{z+1} - \frac{2}{5}\frac{z}{z+3}$$

Taking z^{-1} on both sides,

$$z^{-1}[F(z)] = \frac{1}{15}z^{-1}\left(\frac{z}{z-2}\right) + \frac{1}{3}z^{-1}\left(\frac{z}{z+1}\right) - \frac{2}{5}z^{-1}\left(\frac{z}{z+3}\right)$$

$$y_n = \frac{1}{15}2^n + \frac{1}{3}(-1)^n + -\frac{2}{5}(-3)^n.$$

2] Solve using z-transforms

$y_{n+2} - 3y_{n+1} - 10y_n = 0$ with $y_0 = 1$, $y_1 = 0$.

$$y_{n+2} - 3y_{n+1} - 10y_n = 0.$$

$$z[y_{n+2}] - 3z[y_{n+1}] - 10z[y_n] = 0.$$

$$z^2F(z) - z^2y_0 - zF(z) - 3[zF(z) - zy_0] - 10F(z) = 0$$

$$z^2F(z) - z^2y_0 - zF(z) - 3zF(z) + 3zy_0 - 10F(z) = 0$$

$$z^2F(z) - z^2 - 3zF(z) + 3z - 10F(z) = 0.$$

$$[z^2 - 3z - 10]F(z) = z^2 - 3z$$

$$F(z) = \frac{z^2-3z}{z^2-3z-10}$$

$$\frac{F(z)}{z} = \frac{z-3}{(z-5)(z+2)} \longrightarrow ①$$

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$$\frac{z-3}{(z+2)(z-5)} = \frac{A}{z-5} + \frac{B}{z+2}$$

$$= \frac{A(z+2)+B(z-5)}{(z-5)(z+2)}$$

$$z-3 = A(z+2)+B(z-5)$$

When $z=5 \Rightarrow 5-3 = A(5+2)+B(5-5)$

$$2 = A(7)$$

$$A = 2/7$$

$z=-2 \Rightarrow -2-3 = A(-2+2)+B(-2-5)$

$$-5 = -B(7)$$

$$B = 5/7$$

$$① \Rightarrow \frac{F(z)}{z} = \frac{2/7}{z-5} + \frac{5/7}{z+2}$$

$$F(z) = \frac{2}{7}\left(\frac{z}{z-5}\right) + \frac{5}{7}\left(\frac{z}{z+2}\right)$$

$$z^{-1}(F(z)) = \frac{2}{7}\, z^{-1}\left(\frac{z}{z-5}\right) + \frac{5}{7}\, z^{-1}\left(\frac{z}{z+2}\right)$$

$$= \frac{2}{7}(5)^n + \frac{5}{7}(-2)^n$$