



DEPARTMENT OF MATHEMATICS

Prove the following Boolean identities

i) $a + (a' \cdot b) = a + b$

ii) $a \cdot (a' + b) = a \cdot b$

iii) $(a \cdot b) + (a \cdot b') = a$

Proof:

i)
$$\begin{aligned} a + (a' \cdot b) &= (a + a') \cdot (a + b) \\ &= 1 \cdot (a + b) \\ &= a + b \end{aligned}$$

ii)
$$\begin{aligned} a \cdot (a' + b) &= (a \cdot a') + (a \cdot b) \\ &= 0 + (a \cdot b) \\ &= a \cdot b \end{aligned}$$

iii)
$$\begin{aligned} (a \cdot b) + (a \cdot b') &= a \cdot (b + b') \\ &= a \cdot (1) \\ &= a \end{aligned}$$

Simplify $a' \cdot b' \cdot c + a \cdot b' \cdot c + a' \cdot b \cdot c'$

Soln.

$$\begin{aligned} &a' \cdot b' \cdot c + a \cdot b' \cdot c + a' \cdot b \cdot c' \\ &= a' \cdot b' \cdot c + a' \cdot b' \cdot c' + a \cdot b' \cdot c \\ &= a' \cdot b' \cdot (c + c') + a \cdot b' \cdot c \\ &= a' \cdot b' \cdot (1) + a \cdot b' \cdot c \\ &= b' \cdot (a' + (a \cdot c)) \\ &= b' \cdot ((a' + a) \cdot (a' + c)) \\ &= b' \cdot [1 \cdot (a' + c)] \\ &= b' \cdot (a' + c) \end{aligned}$$

HW In any BA, ST
 $(a+b)(b+c)(c+a) = (a'+b)(b'+c)(c'+a)$



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Atom:

Let $(B, \wedge, \vee, 0, 1)$ be a BA.

A non zero elt. $a \in B$ is called an atom if it is an immediate successor of zero elt.

ie., $0 \leq b \leq a \Rightarrow b=0$ or $b=a$.

Stone's Theorem:

Let B be a finite BA and A be set of all atoms of B . The B.A. B is isomorphic to the BA $P(A)$, where $P(A)$ is the power set of A .

Corollary:

Every finite B.A. $(B, \wedge, \vee, 0, 1)$ has 2^n elts. for some +ve integer n .



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2 element Boolean Algebra:

+	0	1
0	0	1
1	1	1

·	0	1
0	0	0
1	0	1

$\bar{}$	$\bar{\bar{x}}$
0	1
1	0

 $1+1=1$ $1 \cdot 1=1$
 $1+0=1$ $1 \cdot 0=0$
 $0+1=1$ $0 \cdot 1=0$
 $0+0=0$ $0 \cdot 0=0$
 $a+1=1$ $a+a=a$
 $a \cdot 0=0$ $a \cdot a=a$

1. prove that $a+ab=a$

sol:

LHS $a+ab = a(1+b)$ distributive law
 $= a(1)$
 $a+ab = a$

2. $a+\bar{a}b = a+b$

sol:

LHS, $a+\bar{a}b = a+b$
 $a+\bar{a}b = a+ab+\bar{a}b$ from $a = a+ab$
 $= a+b(a+\bar{a})$
 $= a+b(1)$
 $a+\bar{a}b = a+b$

3. $(a+b)(a+c) = a+bc$

sol:

LHS $(a+b)(a+c) = a^2+ac+ba+bc$
 $= a+ac+ba+bc$
 $= a(1+c)+ba+bc$
 $= a(1)+ba+bc$
 $= a+ba+bc$ (or) $a+b$
 $= a(1+b)+bc$
 $= a(1)+bc = a+bc$



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In any Boolean Algebra, show that

$$(a+b')(b+c')(c+a') = (a'+b)(b'+c)(c'+a)$$

Proof:

LHS

$$(a+b')(b+c')(c+a')$$

$$= (ab+ac'+b'b+b'c')(c+a')$$

$$= (abc+aba'+acc'+ac'a'+b'bc+b'ba'+b'c'c'+b'c'a')$$

$$= abc+0+0+0+0+0+0+b'c'a' =$$

[∵ $aa' = 0$,
 $bb' = 0$,
 $cc' = 0$]

$$= abc+a'b'c'$$

RHS.

$$(a'+b)(b'+c)(c'+a)$$

$$= (a'b'+a'c+bb'+bc)(c'+a)$$

$$= a'b'c'+a'b'a+a'ce'+a'ca+bb'c'+bb'a'+bcc'+bca$$

$$= a'b'c'+0+0+0+0+0+0+bca$$

$$= abc+a'b'c'$$

$$(a+b')(b+c')(c+a') = (a'+b)(b'+c)(c'+a)$$

In a Boolean Algebra, prove that

(i) $a \cdot a = a$ and $a+a = a$

(ii) $a \cdot 0 = 0$ and $a+1 = 1$

Proof:

(i) $a \cdot a = a$
Now
 $a = a \cdot 1$



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$$= a \cdot (a+a')$$
$$= (a \cdot a) + (a \cdot a')$$
$$= a \cdot a + 0$$
$$a = a \cdot a$$
$$\boxed{a \cdot a = a}$$

Take dual on both sides

$$\boxed{a+a = a}$$

(ii) $a \cdot 0 = (a \cdot 0) + 0$

$$= (a \cdot 0) + (a \cdot a')$$
$$= a \cdot (0+a')$$
$$= a \cdot 0$$
$$\boxed{a \cdot 0 = 0}$$

Take dual on both sides

$$\boxed{a+1 = 1}$$

Evaluate the expression $x = a \cdot [(b+c) + \bar{d}]$ for
 $a=0, b=0, c=1$ & $d=1$.

Sol:

$$x = 0 \cdot [(0+1) + \bar{1}]$$
$$= 0 \cdot [1+0]$$
$$= 0 \cdot 1$$
$$= 0 \cdot 0$$
$$\boxed{x = 0}$$



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Reduce the expression

(i) $a \cdot \bar{a}b$

$$a \cdot \bar{a}b = 0$$

$$\therefore a \cdot \bar{a} = 0$$

(ii) $a(a+c)$

$$a(a+c) = aa+ac$$

$$= a+ac$$

$$= a(1+c) = a(1) = a$$

(iii) $x(y+z)(x+y+z)$

$$x(y+z)(x+y+z) = (xy+zx)(x+y+z)$$

$$= (yz+zx)(x+y+z)$$

$$= z(y+1)(x+y+z)$$

$$= z \cdot (1)(x+y+z)$$

$$= z(x+y+z)$$

$$= zx+zy+zz$$

$$= zx+zy+z$$

$$= z(x+y+1)$$

$$= z(1)$$

$$= z$$

$$\boxed{x(y+z)(x+y+z) = z}$$

Absorption law in Boolean Algebra.

Statement:

If a and b are two elements of boolean algebra, prove that

$$a + (a \cdot b) = a$$

$$a \cdot (a + b) = a$$



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proof:

now

$$a + (a \cdot b) = (a \cdot 1) + (a \cdot b)$$

$$= a(1+b)$$

$$= a \cdot 1$$

$$= a$$

and

$$a \cdot (a+b) = (a+a) \cdot (a+b)$$

$$= a \cdot a + a \cdot b + a \cdot a + a \cdot b$$

$$= a + a \cdot b + a + a \cdot b$$

$$= a(1+b) + a(1+b)$$

$$= a(1) + a(1)$$

$$= a + a$$

$$= a$$



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sub boolean Algebra

Let $(B, \wedge, \vee, -, 0, 1)$ be a boolean algebra and $S \subseteq B$. If S contains the elements 0 and 1 and it is closed under the operations \wedge, \vee and $-$, then $(S, \wedge, \vee, -, 0, 1)$ is called sub boolean algebra.

prove that D_{110} , the set of all the divisors of the integer 110, is a boolean algebra and find all sub algebras.

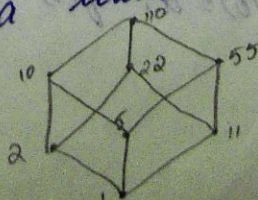
Sol:

$$D_{110} = \{1, 2, 5, 10, 11, 22, 55, 110\}$$

since D satisfies reflexive, antisymmetric, transitive property, D is the partial order relation on D_{110} . (D_{110}, D) it is a poset.

Here $a \wedge b = \text{GLB}(a, b)$
 $a \vee b = \text{LUB}(a, b) \quad \forall a, b \in D_{110}$

(D_{110}, \wedge, \vee) is a lattice. Its hasse diagram is





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Here least element (0 element) is 1
 Greatest element (1 element) is 110

Here each and every element has a complement
 \therefore It is complemented lattice.

from the Hasse diagram it is clear that,
 It is a distributive lattice

(D_{110}, D) is a boolean Algebra.

the subboolean Algebra's are

1. $\{0, 1\} = \{1, 110\}$
2. $\{1, 2, 5, 10, 11, 22, 55, 110\}$
3. $\{a, a', 0, 1\}, \forall a \in S.$

$LCM(1, 110) = 110$
 $GCD(1, 110) = 1$
 $1' = 110$
 $2' = 55$
 $5' = 22$
 $11' = 10$