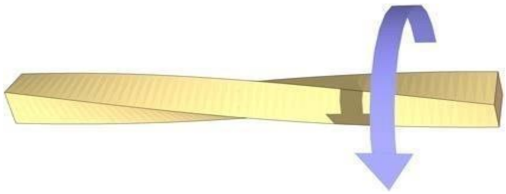
TORSIONAL STRESS AND DEFORMATION

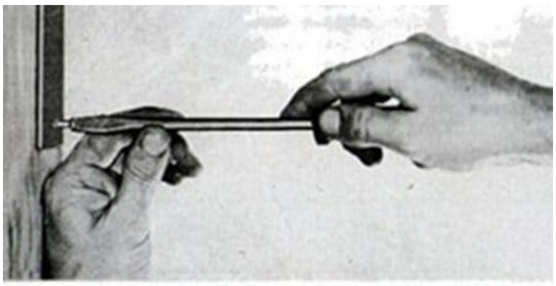
Torsional Stress and Deformations

Shear stress is produced about a longitudinal axis of a structural member by theapplicationoftwistingcoupletotheendofthestructuralmemberisknownas



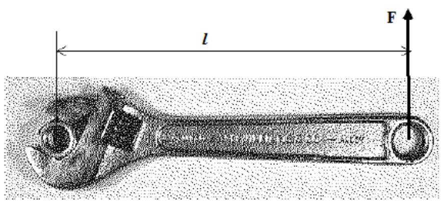
torsional stress. Torsion is the twisting of a straight bar when it is loaded by twisting couple or torque. It tends to produce rotation about the longitudinal axes of the bar.

For instant, when we turn a screw driver to produce torsion our hand applies torque "*T*" to the handle and twist the shank of the screw driver.



TwistingCouple(Torque)

The twisting of a structural member about its longitudinal axis by two equalsand opposite torques is expressed through a certain angle is called twisting couple. Thestressisproducedinthisprocessisnottensileorcompressive,itissaidtobe



shearingorshearstress. Thestrainismeasuredbyanangleinunitofradians.

The simpleexampleisthatofusingawrenchtotightenanutonaboltasshown ingivenfigure. Ifthebolt, wrench,andforceareallperpendiculartoone another.

TWISTINGCOUPLE ONA WIRE

If we have a wire orcylinder, clamped at one end, and twisted at the other through an angle θ, about its axis, it is said to be under tension, due to the elasticity of the material of the wire or the cylinder, a restoring couple is set up in it, equal and opposite to the twisting couple.

Consider a cylindrical wire of length l and radius a. The cylindrical wire is clamped to a fixed support. This wire is made up of a number of cylindrical tubes (co-axial) whose radii vary from zero to „r‟. Let us consider one such cylinder, as shown in fig (b) with radius „x‟ and thickness „dx‟.

Let AB be a line on the elementary tube which is parallel to the axis of the tube. Consider a couple applied at the bottom end of the wire, which results in twisting of the wirethroughanangle θ.Inthetwistedstate,thepositionofABwillbetakenasAB‟as shown in fig 1.6.1

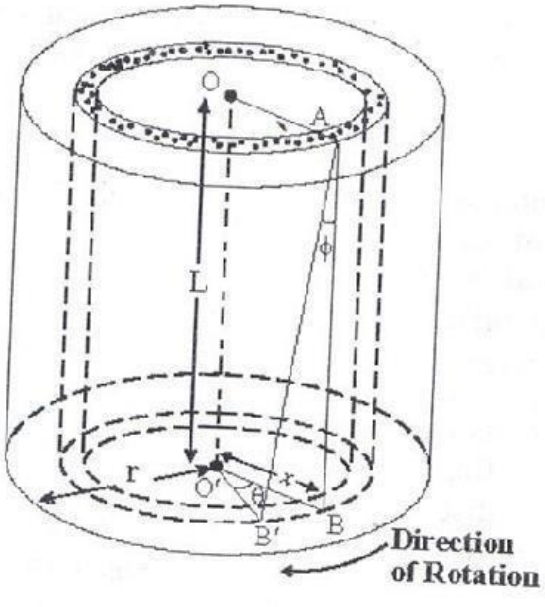


Fig (a)

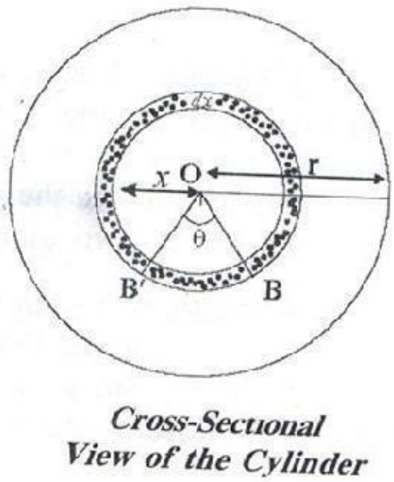


Fig (b)

Fig1.6.1Twistingcoupleon awire.

The displacement decreases, as the move from the rim of the cylinder to the Centre. At the Centre, it will become zero. This means that shearing strain is maximum at the rim and minimum at the Centre.

Consider a hollow cylinder along the plane AB and flattened out. Therefore, we get a rectangle OABO" before twisting and OABB" after twisting. The angle through which this the hollow cylinder is sheared.

Fromthecrosssectionalviewofthecylinder

$BB' \quad x$ …………………. (2)

Fromequations(1)and(2),weget,

$l \quad x$

Shearingstrain $= \frac{x}{l}$ ………………………………………………………………(3)

Weknow that,

Rigiditymodulus(n)= $\frac{shearingstress}{shearingstrain}$

Shearingstress =n Shearingstrain

$n\frac{x}{l}$ ……………………………………………………………… (4)

Lettheareaoftheelementarytube=$2xdx$

Theshearingforceonthisarea =Shearingstress Area

$n\frac{x}{l} \; 2 \; x\,dx$

$\frac{2nx^2dx}{l}$ ………………. (5)

Themomentofforceabouttheaxisofthe wire $\frac{2nx^2dx}{l} \; x$

$\frac{2nx^3dx}{l}$ ……….. (6)

The twisting couple applied to the whole wire can be obtained by integrating equation 6

Between the limit $X\,0 \text{ and } x\,r$

$$\int_0^r \frac{2nx^3dx}{l}$$

$$= \frac{2n^r}{l} \int_0 x^3 dx$$

$$= \frac{2n}{l} \left. \frac{x^4}{4} \right|_0^r$$

$$C = \frac{1}{2} \frac{n\pi r^4 \theta}{l} \qquad \ldots\ldots\ldots\ldots\ldots (7)$$

If θ=1radian

The twisting couples per unit angular twist of the wire

$$C = \frac{1}{2} \frac{n\pi r^4}{l}$$