



## DEPARTMENT OF MATHEMATICS

### UNIT-IV FOURIER TRANSFORM

#### FOURIER TRANSFORM

Fourier transform of  $f(x)$  is

$$F(s) = F(f(x)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx.$$

$$(e^{isx} = \cos sx + i \sin sx)$$

Inversion formula for Fourier transform  $F(s)$  is

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) e^{-isx} ds.$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(f(x)) e^{-isx} ds.$$

$$(e^{-isx} = \cos sx - i \sin sx)$$

Parseval's Identity :

If  $F(s)$  is the Fourier transform of  $f(x)$ . Then

$$\int_{-\infty}^{\infty} [f(x)]^2 dx = \int_{-\infty}^{\infty} [F(s)]^2 ds$$



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Q Find the Fourier transform of  $f(x) = \begin{cases} x & \text{if } |x| \leq a \\ 0 & \text{if } |x| > a \end{cases}$ .

Soln: The gn. function can be written as .

$$f(x) = \begin{cases} x & \text{if } -a \leq x \leq a \\ 0 & \text{if } a < x < \infty \text{ or } -\infty < x < -a \end{cases}$$

Wk7  $f(s) = F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$

$$F[f(x)] = \frac{1}{\sqrt{2\pi}} \left[ \int_{-\infty}^{-a} 0 dx + \int_{-a}^a x e^{isx} dx + \int_a^{\infty} 0 dx \right]$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-a}^a x e^{isx} dx$$

$$= \frac{1}{\sqrt{2\pi}} \left[ x \frac{e^{isx}}{is} - \frac{e^{isx}}{(is)^2} \right]_a^{-a}$$

$$= \frac{1}{\sqrt{2\pi}} \left[ a \frac{e^{isa}}{is} - \frac{e^{isa}}{(is)^2} - \left[ -a \frac{e^{-isa}}{is} + \frac{e^{-isa}}{(is)^2} \right] \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[ a \frac{e^{ias}}{is} - \frac{e^{ias}}{(is)^2} + a \frac{e^{-ias}}{is} + \frac{e^{-ias}}{(is)^2} \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[ \frac{a}{is} [e^{ias} + e^{-ias}] - \frac{1}{(is)^2} [e^{ias} - e^{-ias}] \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[ \frac{a}{is} 2 \cos as - \frac{1}{(is)^2} 2i \sin as \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[ \frac{a}{is} 2 \cos as + \frac{1}{s^2} 2i \sin as \right]$$

$$= \sqrt{\frac{2}{\pi}} \left[ \frac{a}{is} \cos as + \frac{i \sin as}{s^2} \right]$$

$$= \sqrt{\frac{2}{\pi}} i \left[ \frac{a}{i^2 s} \cos as + \frac{\sin as}{s^2} \right]$$



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$$= i \sqrt{\frac{2}{\pi}} \left[ \frac{\sin as}{s^2} - \frac{a \cos as}{s} \right]$$
$$= i \sqrt{\frac{2}{\pi}} \left[ \frac{\sin as - as \cos as}{s^2} \right].$$