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# DEPARTMENT OF MATHEMATICS UNIT-IV FOURIER TRANSFORM

SINE TRANSFORM:

The following transform q a function f(m), or  $n \ge 0$  is defined as  $F_S(S) = F_S[F_S(m)] = \sqrt{\frac{n}{n}} \int_{-\pi}^{\infty} f(n) \sin n \, dn$ The fiverine following sine transform of  $F_S(S)$  is defined as  $f(m) = F^{-1}[F_S(S)] = \sqrt{\frac{n}{n}} \int_{-\pi}^{\infty} f_S(S) \sin n \, ds$ .

Passeval's fdentity:  $\Re_F f_S(S) = 1$  The Following transform g = f(m) then  $\int_{-\pi}^{\infty} f_S(n) f(n) \, dn = \int_{-\pi}^{\infty} f_S(S) f(n) \, ds$ .

COSINE TRANSFORM

The focuse conneteansform of a function f(n), or  $n < \infty$  is defined as  $F_c(s) = f_c(f_c(n)) = \sqrt{\frac{2}{n}} \int_{-\infty}^{\infty} f(n) \cos sn \, dn$ .

The Sovere Fourier work transform of  $F_c(s)$  is defined as  $f(n) = F^{-1}[F_c(s)] = \sqrt{\frac{2}{n}} \int_{-\infty}^{\infty} f_c(s) \cos sn \, ds$ .

If  $F_c(s)$  is the fourier transform of f(n), then passevals of f(n) is  $\int_{-\infty}^{\infty} f_c(n) f^2 \, dn = \int_{-\infty}^{\infty} f_c(s) f^2 \, ds$ 





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1) Find the fourier sine teansform of ten defined by

$$f(n) = \begin{cases} 1 & 3 & 0 < n < 1 \\ 0 & 3 & n > 1 \end{cases}$$

$$= soln! \quad \text{WhT} \quad f_s(s) = \sqrt{\frac{2}{11}} \int_{0}^{1} f(n) \sin n \, dn$$

$$= \sqrt{\frac{2}{11}} \int_{0}^{1} \sin sn \, dn$$

$$= \sqrt{\frac{2}{11}} \left[ \frac{1 - uss}{s} \right]_{0}^{1}$$

$$= \sqrt{\frac{2}{11}} \left[ \frac{1 - uss}{s} \right]_{0}^{1}$$

Problem For the following sine transform of 
$$r$$
 .

Solve For (s) =  $\sqrt{\frac{a}{\pi}} \int_{0}^{\infty} f(m) \sin n \, dn$ 

$$= \sqrt{\frac{a}{\pi}} \int_{0}^{\infty} \frac{\sin n}{n} \, dn$$

Putting  $0 = \sin \Rightarrow d0 = \sin n$ 

$$= \sqrt{\frac{a}{\pi}} \int_{0}^{\infty} \frac{\sin n}{n} \, ds$$





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3) Find the fouries cosme transform of 
$$2e^{-3\eta} + 3e^{-2\eta}$$
.

Soln:

What  $fc(s) = \sqrt{\frac{2}{11}} \int_{0}^{\infty} \frac{1}{3} (m) \cos 3n \, dn$ 

$$= \sqrt{\frac{2}{11}} \int_{0}^{\infty} (2e^{-3\eta} + 3e^{-2\eta}) \cos sn \, dn$$

$$= \sqrt{\frac{2}{11}} \left[ 2 \left[ \frac{3}{8^{2}+9} \right] + 3 \left[ \frac{2}{8^{2}+4} \right] \right]$$

$$= \sqrt{\frac{2}{11}} \left[ \frac{6}{6^{2}+9} + \frac{6}{8^{2}+11} \right]$$

4) Find the Fourier cosine transform of 
$$f(n) = \int \cos n$$
, if oraca sotn: White  $f(n) = \int \frac{1}{\pi} \int \frac{1}{\pi}$ 





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5> Find The Fourier sine & cosine teansform  $g e^{-aa}$  and deduce that inverse fourier teansform & paesevals robinkty soln:

Sine teansform:

White  $f(s) = \sqrt{\frac{a}{\pi}} \int_{0}^{\infty} g(n) \sin sn \, dn$   $= \sqrt{\frac{a}{\pi}} \int_{0}^{\infty} e^{-an} \sin sn \, dn$   $= \sqrt{\frac{a}{\pi}} \int_{0}^{\infty} e^{-an} \sin sn \, dn$ 





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Sinverse Transform:

Whit 
$$f(m) = F^{-1}[F_s(s)] = \sqrt{\frac{2}{\pi}} \int_{0}^{\infty} F_s(s) g_{n}^{s} s_{n} ds$$

$$f(m) = \sqrt{\frac{2}{\pi}} \int_{0}^{\infty} \frac{g}{g_{7}^{2} q^{2}} f(s) s_{n}^{s} s_{n} ds$$

$$e^{-an} = \sqrt{\frac{2}{\pi}} \sqrt{\frac{g}{m}} \frac{g}{g_{7}^{2} q^{2}} f(s) s_{n}^{s} s_{n} ds$$

$$\frac{\pi}{a} e^{-an} = \int_{0}^{\infty} \frac{g}{g_{7}^{2} q^{2}} f(s) s_{n}^{s} s_{n} ds$$

$$\int_{0}^{\infty} e^{-an} \int_{0}^{\infty} dn = \int_{0}^{\infty} \sqrt{\frac{g}{n}} \left[ \frac{g}{g_{7}^{2} q^{2}} \right]^{2} ds$$

$$\int_{0}^{\infty} e^{-2an} dn = \frac{2}{\pi} \int_{0}^{\infty} \frac{g}{g_{7}^{2} q^{2}} ds$$

$$\frac{e^{-2an}}{-2a} \int_{0}^{\infty} \frac{1}{3a} = \frac{2}{\pi} \int_{0}^{\infty} \frac{g}{g_{7}^{2} q^{2}} ds$$

$$\frac{\pi}{4a} = \int_{0}^{\infty} \frac{g}{g_{7}^{2} q^{2}} ds$$





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Cosine Teansferm:

Whi 
$$f_c(s) = \sqrt{\frac{2}{11}} \int_0^{\infty} f(n) \cos sn dn$$

$$= \sqrt{\frac{2}{11}} \int_0^{\infty} e^{-an} \cos sn dn$$

$$= \sqrt{\frac{2}{11}} \int_0^{\infty} e^{-an} \cos sn dn$$

Invesce Teansform!

WHAT 
$$\sqrt{m} = F^{-1}[F_c(s)] = \sqrt{\frac{a}{\pi}} \int_{0}^{\infty} F_c(s) \cos sn \, ds$$

$$\sqrt{m} = \sqrt{\frac{a}{\pi}} \int_{0}^{\infty} \frac{a}{s^2 + a^2} \int_{0}^{\infty} ss \, sn \, ds$$

$$e^{-an} = \sqrt{\frac{a}{\pi}} \cdot \sqrt{\frac{a}{\pi}} \int_{0}^{\infty} \frac{a}{s^2 + a^2} \cos sn \, ds$$

$$1 = \frac{a}{\pi} \int_{0}^{\infty} \frac{a}{s^2 + a^2} \cos s(s) \, ds$$

$$\frac{\pi}{2} = \int_{0}^{\infty} \frac{a}{s^2 + a^2} \, ds$$





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Passeval's elentity:

Whit 
$$\int_{0}^{\infty} \left[f(n)\right]^{2} dn = \int_{0}^{\infty} \left[F_{c}(s)\right]^{2} ds$$

$$\int_{0}^{\infty} \left[e^{-\alpha n}\right]^{2} dn = \int_{0}^{\infty} \sqrt{\frac{\alpha}{n!}} \frac{\alpha}{s^{2}+\alpha^{2}} ds$$

$$\int_{0}^{\infty} e^{-2\alpha n} dn = \frac{2}{n!} \int_{0}^{\infty} \frac{\alpha}{s^{2}+\alpha^{2}} ds$$

$$\frac{e^{-2\alpha n}}{2\alpha} \int_{0}^{\infty} \frac{1}{2\alpha} ds = \frac{2}{n!} \int_{0}^{\infty} \left[\frac{\alpha}{s^{2}+\alpha^{2}}\right]^{2} ds$$

$$\frac{n!}{n!} = \int_{0}^{\infty} \left[\frac{\alpha}{s^{2}+\alpha^{2}}\right]^{2} ds$$