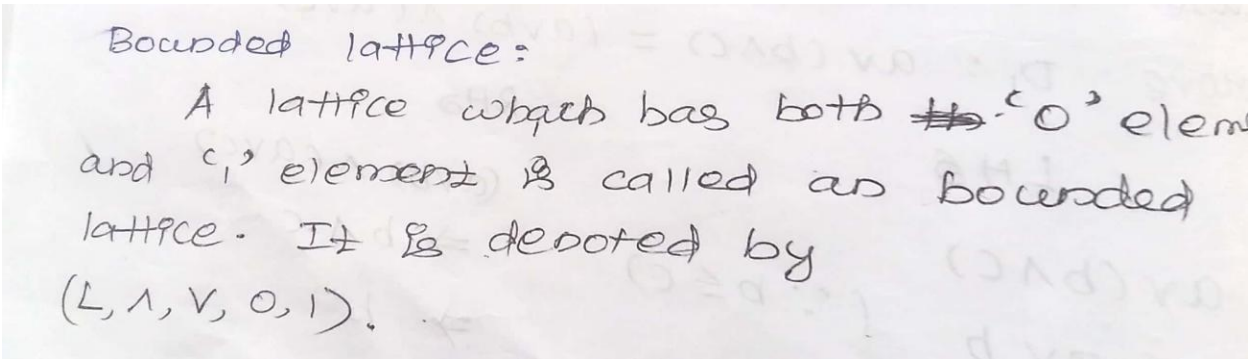
UNIT 5– LATTICES AND BOOLEAN ALGEBRA Lattices as algebraic systems



Complement of an element:

In a bounded lattice $(L, \wedge, \vee, 0, 1)$, an elt. $b \in L$ is called a complement of $a \in L$, if $a \wedge b = 0$, $a \vee b = 1$

Complemented lattice:

A lattice $(L, \wedge, \vee, 0, 1)$ is said to be complemented lattice if every elt. of L has atleast one complement.

Complete lattice:

A lattice $(L, \wedge, \vee)$ is said to be complete lattice if every non empty subsets of L has both GLB & LUB.

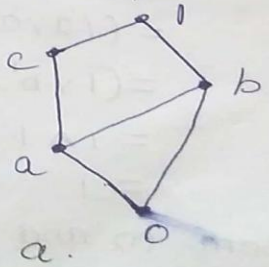
Eg: $(P(A), \subseteq)$

Modular lattice:

A lattice $(L, \wedge, \vee)$ is said to be modular lattice, if it satisfies the following condition

M_1: If $a \leq c$ then $a \vee (b \wedge c) = (a \vee b) \wedge c$, $\forall a, b, c \in L$.

1. Check the given lattice is the complemented lattice or not.



Now

$a \wedge b = a \neq 0$

$a \vee b = b \neq 1$

$\Rightarrow$ b is not the complement of a.

and $b \wedge c = a \neq 0$

$b \vee c = 1$

$\Rightarrow$ b is not the complement of c.

$\therefore$ It is not a complemented lattice.